EVALUATING ENVIRONMENTAL JOINT EXTREMES FOR THE OFFSHORE INDUSTRY

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The Problem

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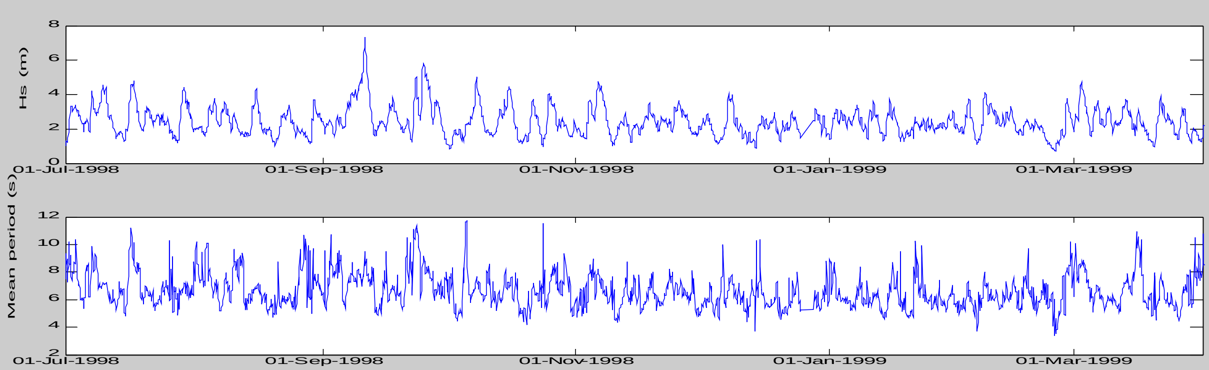
Motivation - General

<u>Environmental Design criteria</u>

- Design of offshore structures requires environmental parameters to be specified with very low probabilities of occurrence
- Design codes stipulate that offshore structures should be designed to exceed specific levels of reliability, expressed in terms of an annual probability of failure or return-period.
- Application is focused on structural loading not environment
- Many physical systems respond to environmental conditions in a manner that cannot be represented by a single variable or parameter
    - pitch of a vessel is as much a function of the wave period or wave length as it is of the wave height.
    - extremes values of Hs and associated values for Tp at the extreme value of Hs.

Motivation – Design Philosophy

- **Goal:** design offshore facility to withstand extreme environmental conditions that will occur during its lifetime with "optimum" risk level
  - Weigh consequences of failure against cost of over-designing
  - For facilities with 20-30-year lifetime, generally use 100-year metocean criteria
  - With typical implicit and explicit safety factors, annual probability of failure ≈ 10^{-3} to 10^{-5} (5×10^{-4} for normally unmanned, but 3×10^{-5} for manned)
- **Procedure:** Analyse historical environmental conditions, and assume future exposure will have the same statistics of extremes as past exposure



Motivation – Design Philosophy

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- **Procedure:** Analyse historical environmental conditions, and assume future exposure will have the same statistics of extremes as past exposure

  Global climate change:
  - Effect on extreme conditions is not yet known, and "effect" is generally omitted
  - Assume design safety factors will accommodate any increase in environmental severity that may occur
  - Possible to model non-stationary extremes with time as a covariate

Motivation – Design Philosophy – Platform (steel jacket)

- Conventional - Working Stress Design (API WSD)
  - Every component, i, in the structure should satisfy:

$$R_i / FoS_i > D_i + L_i + E_{100}$$

- R_i Resistance
- D_i, L_i, E_{100} Dead , Live, Environmental loads
- FoS_i Factor of Safety ~ 1.25 – 1.50 (tension, compression, foundation)

- Load-Resistance Factor Design (ISO 19902, API LRFD)

$$R_i / \gamma_R > \gamma_D D_i + \gamma_L L_i + \gamma_E E_{100}$$

  - γ_R Resistance factor (e.g. 1.18 compression)
  - $\gamma_D = 1.1, \gamma_L = 1.1, \gamma_E = 1.35$ Dead , Live, Environmental load factors

- Reliability
  - R, D, L, E treated randomly to calculate prob (load > resistance)

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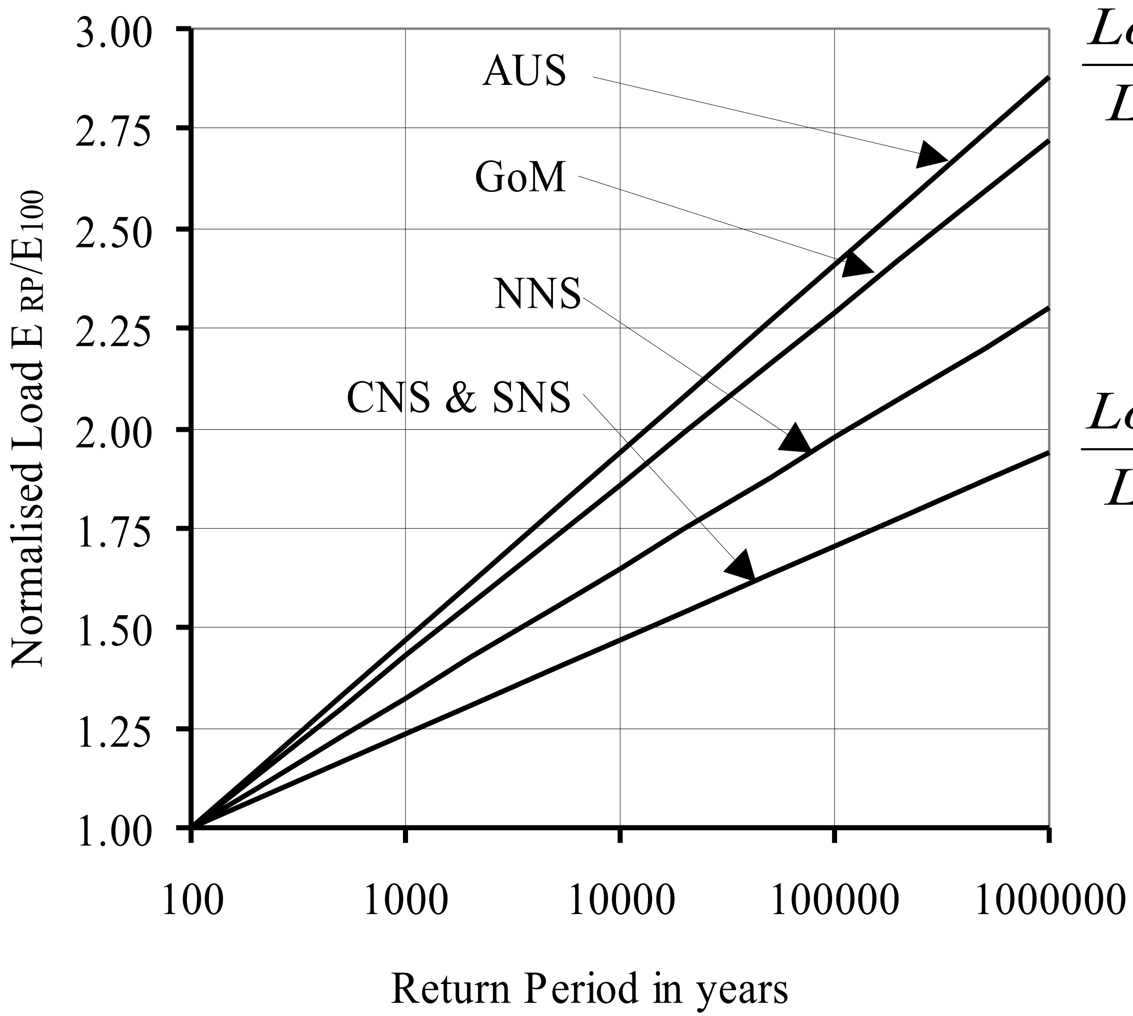
Motivation – Design Philosophy – Platform (steel jacket)



$$\frac{Load_{10000}}{Load_{100}} = 1.90$$

$$\frac{Load_{10000}}{Load_{100}} = 1.45$$

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Motivation – Design Philosophy

- Environmental criteria developed independently for each parameter
  - Hmax (100-year), Ws (100-year), Cs (100-year)
  - Probability of those occurring together << 0.01pa
  - Conservative, and unnecessary cost

- Application of Independent Criteria
  - Typically 100-yr wind, wave and current in the same direction (inline & +/- 30°)
  - Possibly 100-yr wind, 100-yr wave, 10-yr current in the same direction (inline & +/- 30°)
  - Or other combinations, but always from independent metocean conditions

Motivation – Standards & Guidelines

- API Bulletin 2INT-MET – Interim Conditions for Gulf of Mexico

Table 5-1: Factors for Combining Independent Extremes into Load Cases in Deep Water (WD >= 150 m or 492 ft)

Return Period (Years)	10	25	50	100	200	1000	2000	10000
Peak Wave Case:								
Wind Speed	1.00	0.95	0.95	0.95	0.95	0.95	0.95	0.95
Wave Height	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Current (both speed and depth level)	0.80	0.80	0.75	0.75	0.75	0.75	0.75	0.75
Surge	0.90	0.80	0.70	0.70	0.70	0.70	0.70	0.70
Wind Direction from Wave (deg)	–15	–15	–15	–15	–15	–15	–15	–15
Current Direction from Wave (deg)	+15	+15	+15	+15	+15	+15	+15	+15
Peak Wind Case:								
Wind Speed	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Wave Height	1.00	0.95	0.95	0.95	0.95	0.95	0.95	0.95
Current (both speed and depth level)	0.80	0.80	0.75	0.75	0.75	0.75	0.75	0.75
Surge	0.90	0.80	0.70	0.70	0.70	0.70	0.70	0.70
Wind Direction from Wave (deg)	–15	–15	–15	–15	–15	–15	–15	–15
Current Direction from Wave (deg)	+15	+15	+15	+15	+15	+15	+15	+15
Peak Current Case:								
Wind Speed	0.75	0.70	0.70	0.70	0.70	0.70	0.70	0.70
Wave Height	0.75	0.70	0.70	0.70	0.70	0.70	0.70	0.70
Current (both speed and depth level)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Surge	0.90	0.80	0.70	0.70	0.70	0.70	0.70	0.70
Wind Direction from Wave (deg)	0	0	0	0	0	0	0	0
Current Direction from Wave (deg)	+50	+50	+50	+50	+50	+50	+50	+50

NOTE: When factoring surge from Figures 4.5.1-4, 4.5.2-4, 4.5.3-4 and 4.5.4-4, remove the tidal amplitude, factor the surge, then add the tidal amplitude back in

Motivation – Standards & Guidelines

- DNV – Recommended Practice DNV-RP-F109 ON-BOTTOM STABILITY DESIGN OF SUBMARINE PIPELINES

  - The characteristic load condition shall reflect the most probable extreme response over a specified design time period.
  - When detailed information about the joint probability of waves and current is not available, this condition may be approximated by the most severe condition among the following two combinations:

  1. The **100-year** return condition for **waves** combined with the **10-year** return condition for **current**.
  2. The **10-year** return condition for **waves** combined with the 1**00-year** return condition for **current.**

Motivation – Examples

- Hurricane in the Gulf of Mexico / Continental Shelf Australia
  - Local wind-field drives the waves and currents
  - Rapidly changing conditions

    => Probability of experiencing extreme wind, waves and current is high
- West Africa
  - Swells from South Atlantic storms, run normal to coast
  - Currents from ocean circulation, run along coast

    => Probability of experiencing extreme waves and extreme currents is low
- Arabian Gulf
  - Wave extremes due to "Shamal"
  - Currents dominated by tides

    => Probability of experiencing extreme waves and extreme currents is relatively low, but they are largely inline

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Platform Reliability

- Unknown probability of design sea states from combinations of independent criteria
- Solution: Joint criteria with known probability of occurrence
- Environmental Approach
  - Develop joint criteria associated with rare return periods
- Response-based Approach
  - Relies on specification of a response model giving load as a function of environment and a back calculation of the environmental parameters
- Combination of Environmental and Response-based

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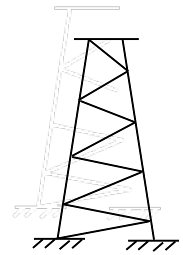
Response-based Approach - Introduction

Environment "perspective" | **Response "perspective"**

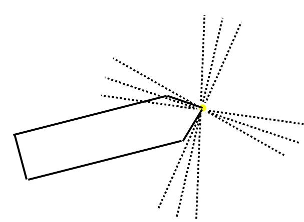
10^{-4} { Wind / Waves / Current / Directionality → **Structural Model** → 10^{-4}

10^{-4} { Wind / Waves / Current / Directionality → (Waves shown in red) → 10^{-4}



Simple Jacket

10^{-4} { **Wind** / **Waves (sea /swell)** / **Current** / **Directionality!** → 10^{-4}



Response-based Approach – Fixed Structures - Outline

- Assumed that a long-term data set of all the required metocean parameters exist: u, a, T, …
  - Usually available from hindcast studies
    - some parameters may need to be calculated from the hindcast data
- Generic Load Model for base shear or mud-line overturning moment is defined

$$E = A_1 u^2 + A_2 u a T \phi \cos\theta + A_3 a^2 \phi^2 \cos\theta_c + A_4 u a^2 \phi \cos\theta_c / T + \ldots$$
$$A_5 a^3 \phi^2 / T + A_6 a^2 \phi^2 T^2 + A_7 W^2 \cos\theta_W$$

a	is the linear crest elevation
ϕ	is the wave spreading factor
T	is the wave zero-crossing period
u	is the depth-average current speed
W	is the one-minute mean wind speed

Response-based Approach – Fixed Structures - Outline

$$E = A_1 u^2 + A_2 u a T \phi \cos\theta + A_3 a^2 \phi^2 \cos\theta_c + A_4 u a^2 \phi \cos\theta_c / T + \ldots$$
$$A_5 a^3 \phi^2 / T + A_6 a^2 \phi^2 T^2 + A_7 W^2 \cos\theta_W$$

θ_c is the angle between mean wave and current directions

θ_W is the angle between mean wave and wind directions

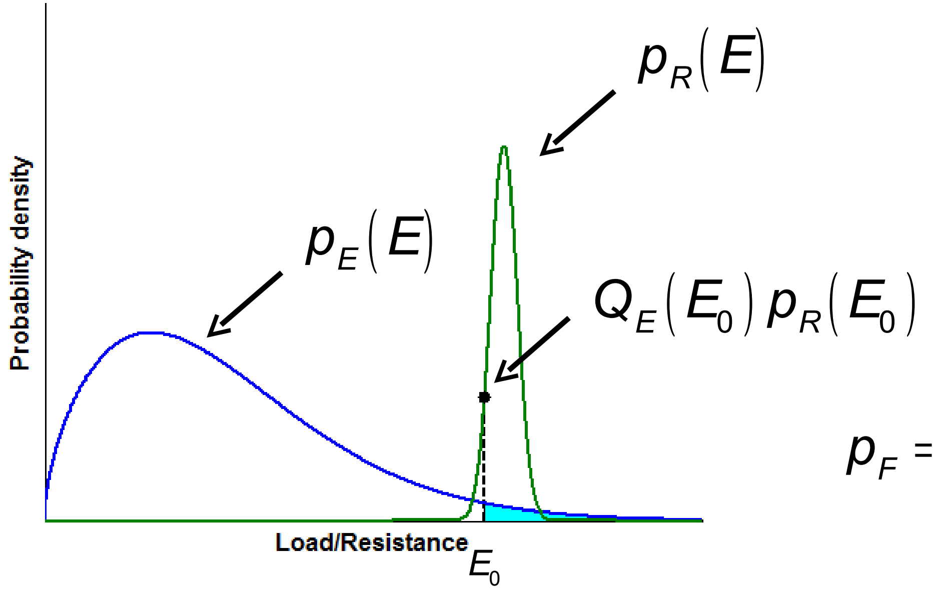
A_i depend on structure and mean wave direction

determined by calibration of large number of conditions with a given wave kinematics and current profile on a one meter diameter column.

- Response model used to establish long data base of E
- Extremal analysis undertaken to determine e.g. E_{100}

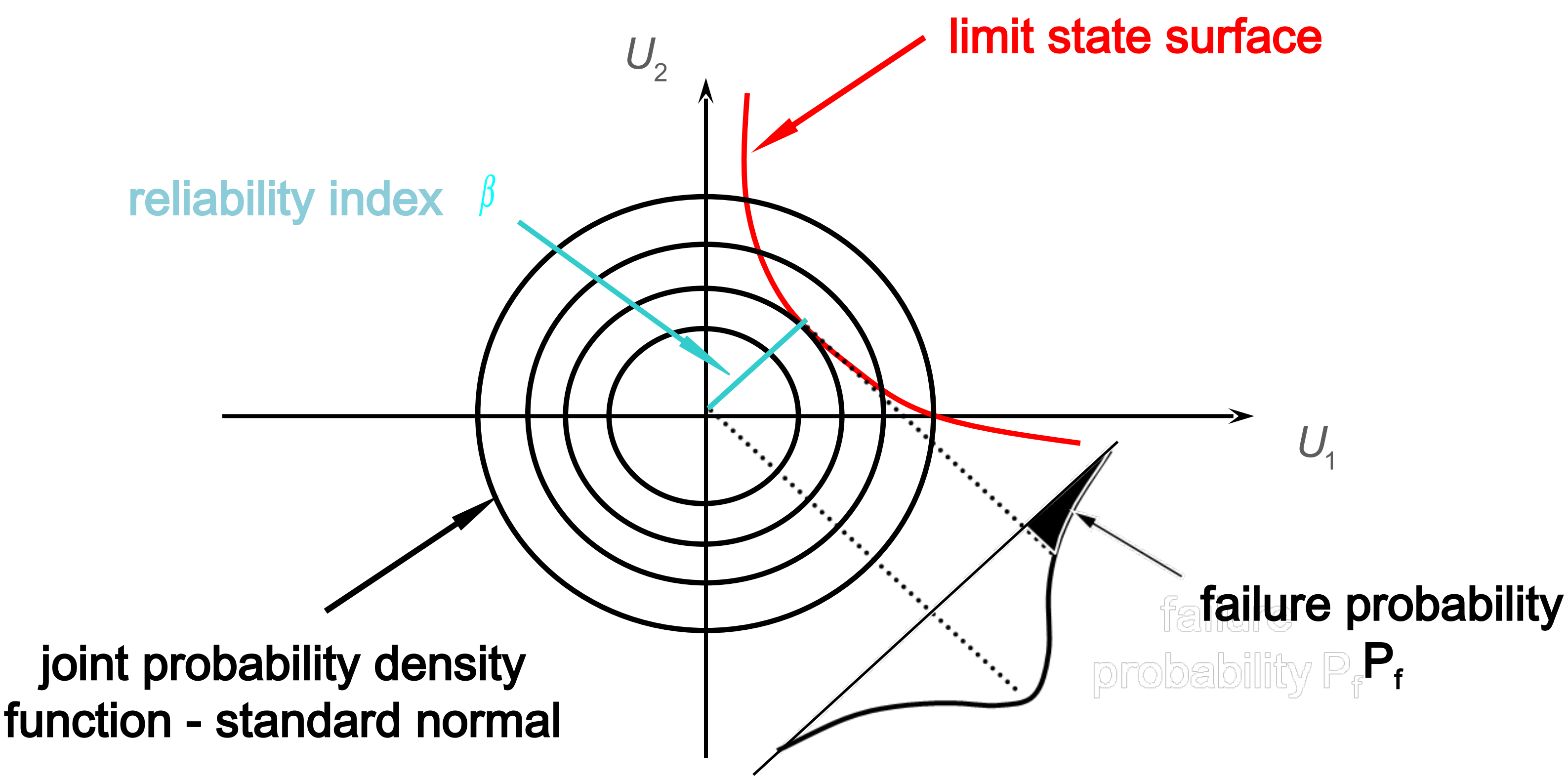
Response-based Approach – Fixed Structures - Outline

- Environmental criteria determined by back-calculation from e.g. E_{100}
  - Difficult and usually involves some assumptions and judgement
    - Choose a dominant parameter such as a_{100}
- Assume that there is a probabilistic model for structural strength or resistance, R, to the environmental load E, failure probability follows



$$p_F = \int Q_E(E)\,p_R(E)\,dE$$

Reliability – FORM

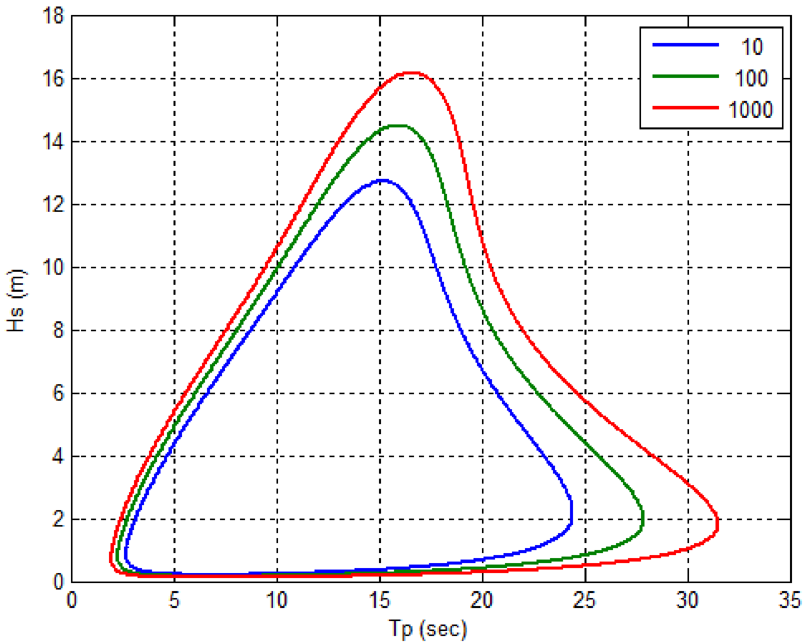


Inverse-FORM - Outline

- **FORM** starts with a failure surface and calculates failure probability
  - Failure surface tangential to the surface at the design point
  - Response for nearby points can be checked to ensure the actual surface is outside the surface
- **IFORM** starts with a failure probability and calculates a design point on an associated failure surface.
- Approach used to produce “environmental contours” – low-probability combinations of environmental parameters

Inverse-FORM - Outline – 2 variate example

- Hs : Weibull $\overset{PIT}{\Rightarrow}$ U_1 : standard Normal
- $Tp \mid Hs$: log-Normal $\overset{PIT}{\Rightarrow}$ U_2 : standard Normal
- Circle $U_1^2 + U_2^2 = \beta^2$ gives constant probability
- $(U_1, U_2) \overset{PIT}{\Rightarrow} (Hs, Tp)$



FORM - Characteristics

- Contour lines of $\mathbf{X}$ are provided, linked to a desired return period
- Assumes we can transform to independent random variables
- Assumes prior knowledge of the distribution of X_1 and $X_2 | X_1$
  - $Tp | Hs$ log-normal often used (largely based on body of distn)
- Explains body of the distribution; not necessarily the tails
- Difficult in practice to extend beyond 2 variates – e.g.

$$p(Hs, Tp, U) = p(U | Hs, Tp)\, p(Tp | Hs)\, p(Hs)$$

- Could ignore U dependence on Hs, Tp and replace with p(U)

Heffernan & Tawn (2004) - Method

- Pairs (X,Y) of random variables
- Assume Gumbel marginal distributions
- Parametric form for conditional distribution of one variable given large value of other $(Y| X = x) = ax + x^b Z$

  $a \in [0,1]$ scale parameter

  $b \in (-\infty, 1]$ shape parameter

  Z random variable, independent of X

  - Z estimated from the sample residuals $\hat{z}_i = \dfrac{y_i - \hat{a}x_i}{x_i^{\hat{b}}}$
- Joint extremes established from simulations

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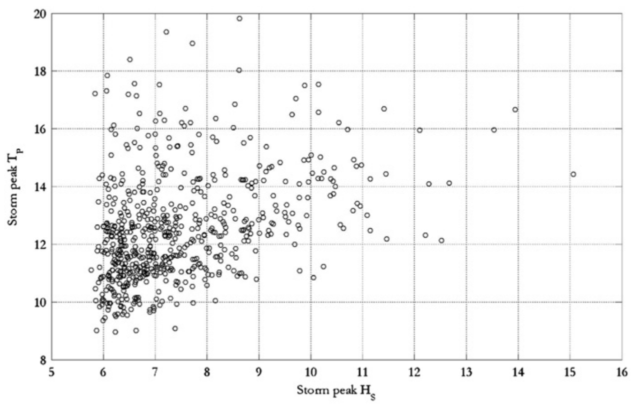
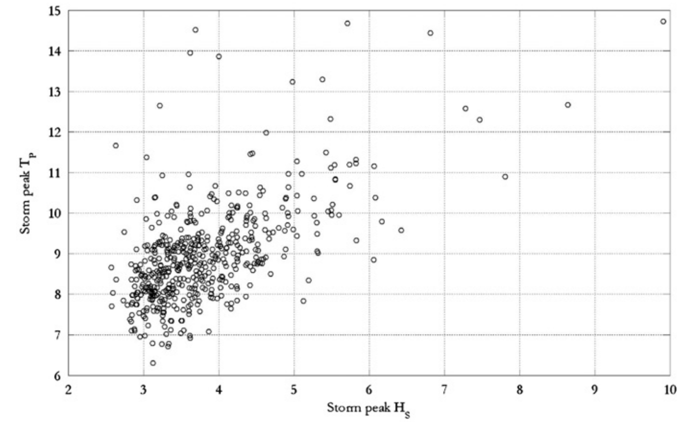
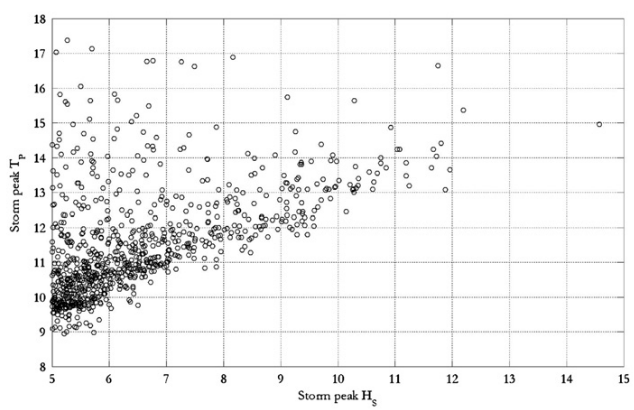
Heffernan & Tawn (2004) - Application



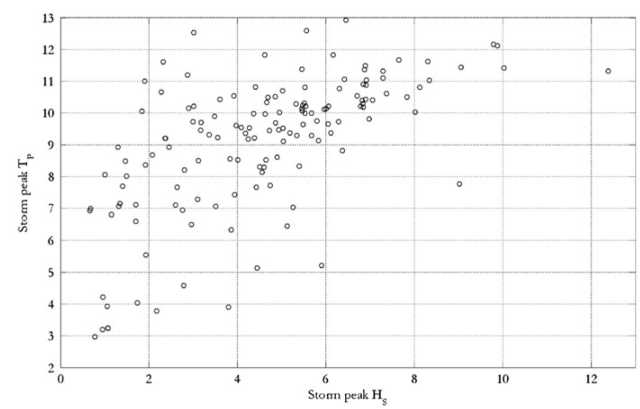
A: NNS - measured



C: GoM - measured



B: NNS - hindcast



D: NWS - hindcast

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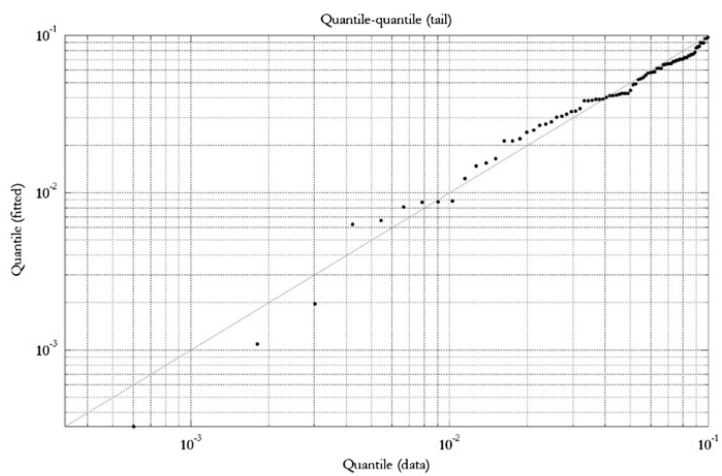
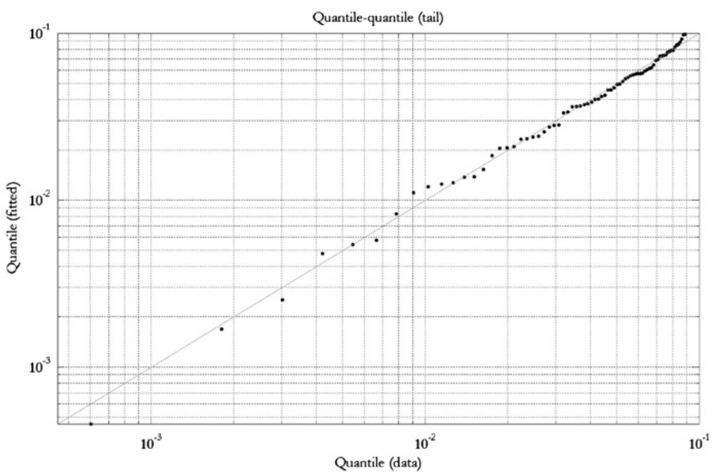
Heffernan & Tawn (2004) - Application

- Marginal GP Fits to storm peak *Hs* and *Tp*
  - QQ Plots indicate GP model goodness of fit



Hs - NNS - measured



Tp - NNS - measured

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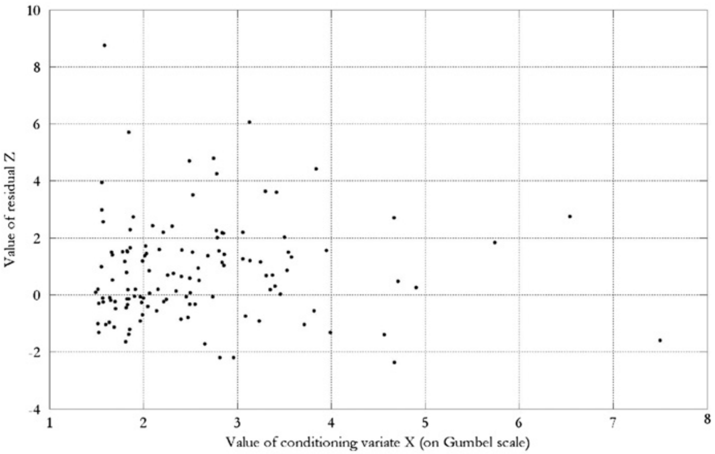
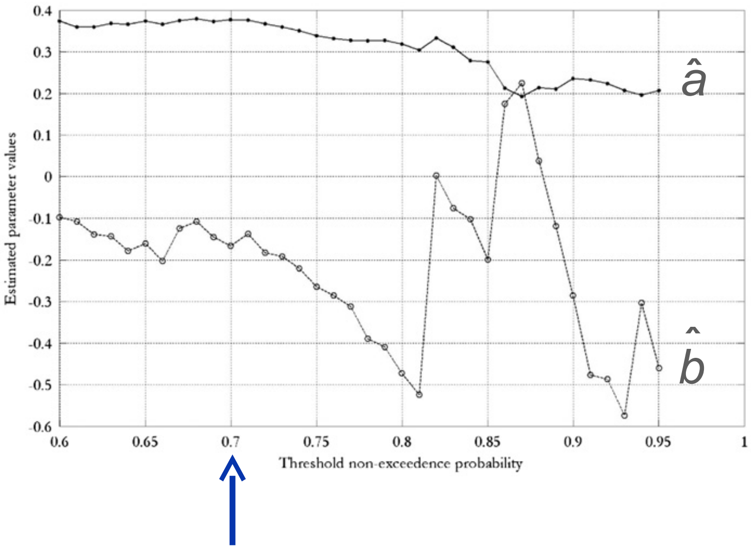
Heffernan & Tawn (2004) - Application

- Diagnostics





Residuals - NNS - measured

Parameters - NNS - measured

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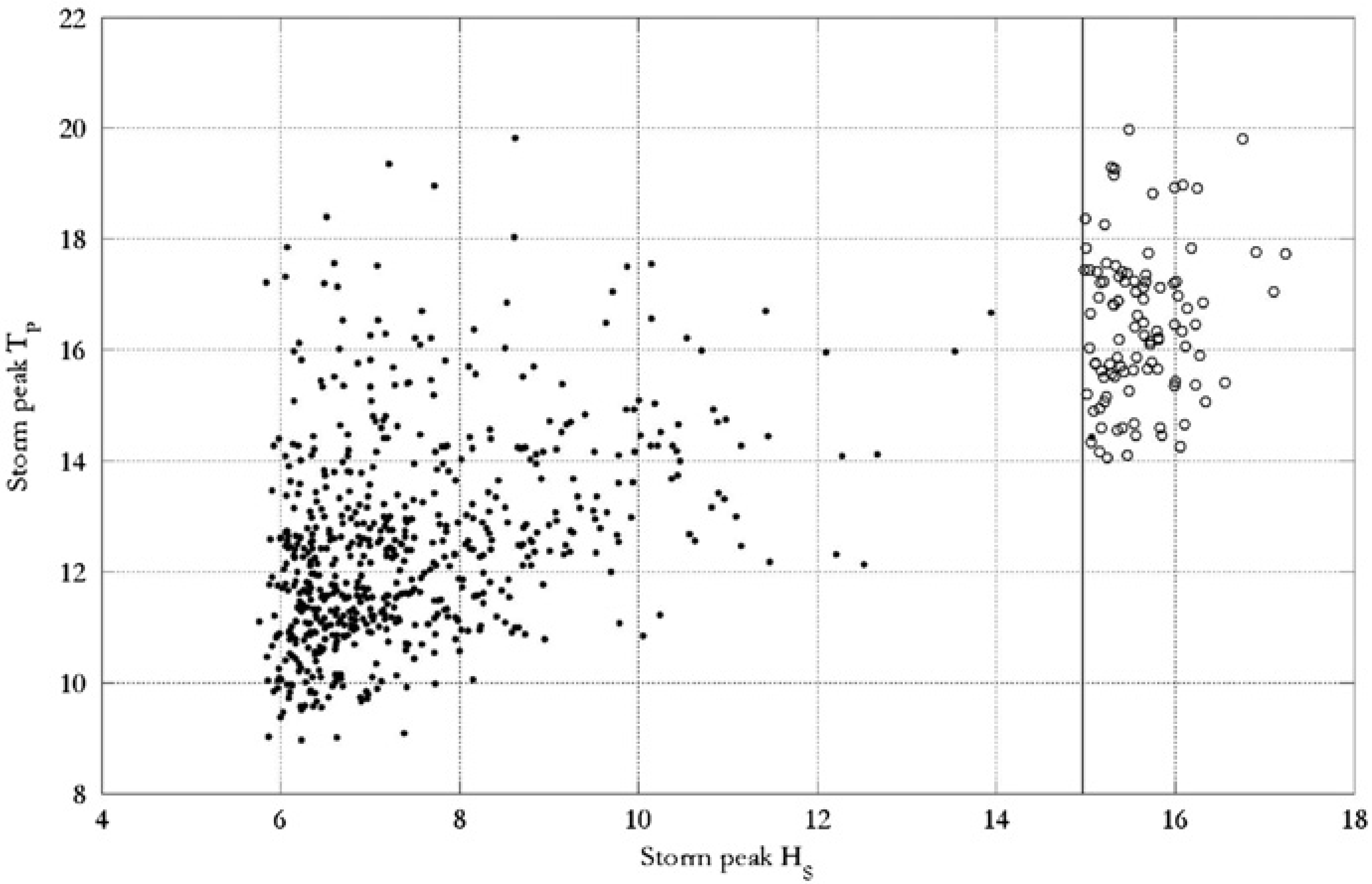
Heffernan & Tawn (2004) - Application

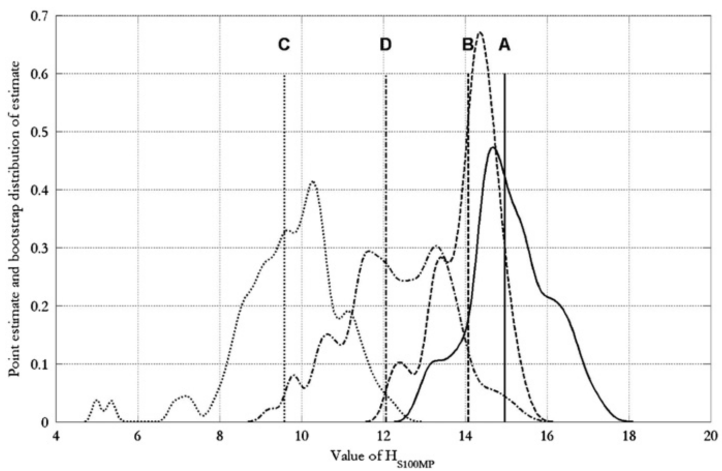
- Illustrative Simulation

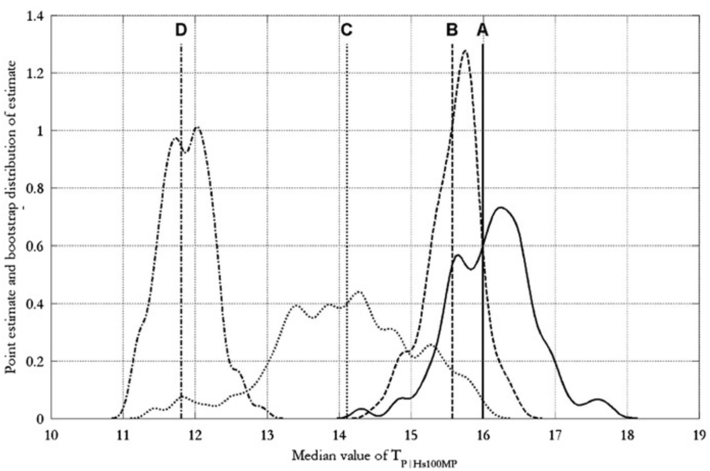


Data and Simulations - NNS - measured

Heffernan & Tawn (2004) - Application

- Point and kernel density distributions (bootstrap) of extreme quantiles





Hs_{100MP} $\quad$ $Tp|Hs_{100MP}$

A: NNS – measured, **B**: NNS – hindcast,**C**: GoM – measured, **D**: NWS - hindcast

Heffernan & Tawn (2004) – Final Points

- Value of conditioning variate must be large for conditional extremes model to apply
- No prior knowledge of form of distribution of X1 and X2|X1 required
- Models tail of distribution using conditional extremes
- Models body of distribution empirically
- Direct estimation of failure probability from simulations
- Easily extended to multi-dimensional case

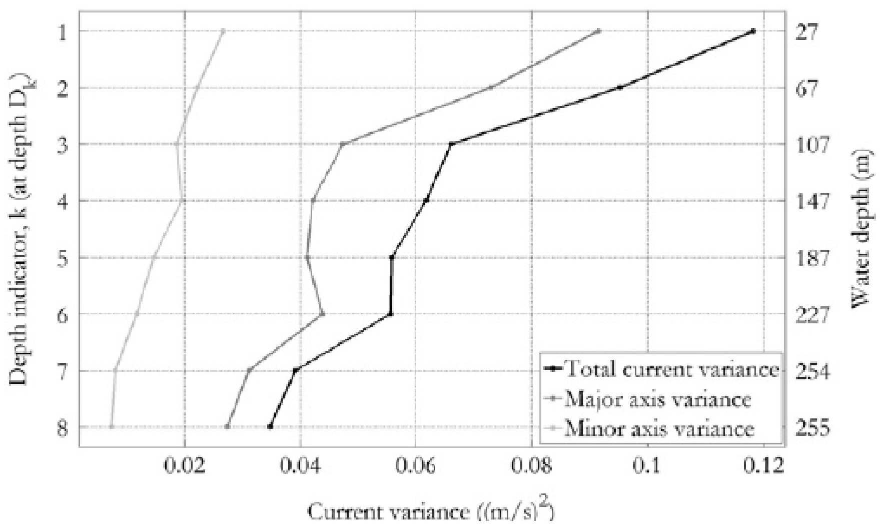
Multi-dimensional Example – Current vector profile

- Current profiles measured on NWS Australia
  - measured with eight single-point current meters
    - Positioned at 27m, 67m, 107m, 147m, 187m, 227m, 254m, and 255m below surface
    - Current velocities 1-minute vector average of measurements sampled at 2 Hz.
    - 2.5 years of measurements
- Current characteristics
  - semi-diurnal tidal component
    - near surface - anti-clockwise rotation; near the seabed – southeast (flood) and northwest (ebb)
  - Wind-forced component
    - Most pronounced under tropical cyclones
    - Monsoonal wind behaviour – near surface currents E/NE in summer, W/SW in winter
  - Regional flow
    - Indian Ocean anti-clockwise gyre, Pacific-Indian Ocean throughflow
  - Inertial currents – 50 hr period.
  - Solitons – short (10min) intense events in record

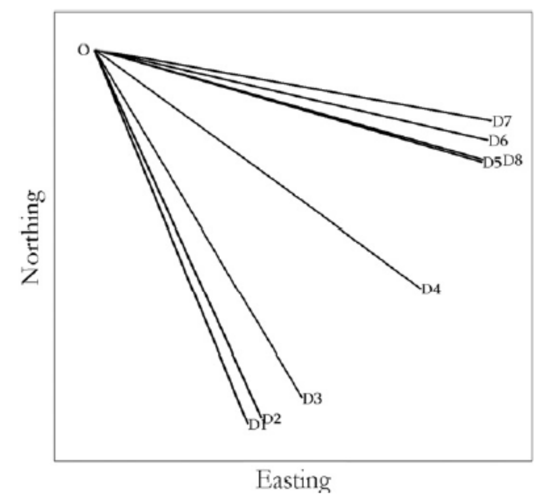
Multi-dimensional Example – Analysis (1)

- Resolve currents into major and minor axes of total current tidal ellipse at each depth

Variance



major axis direction



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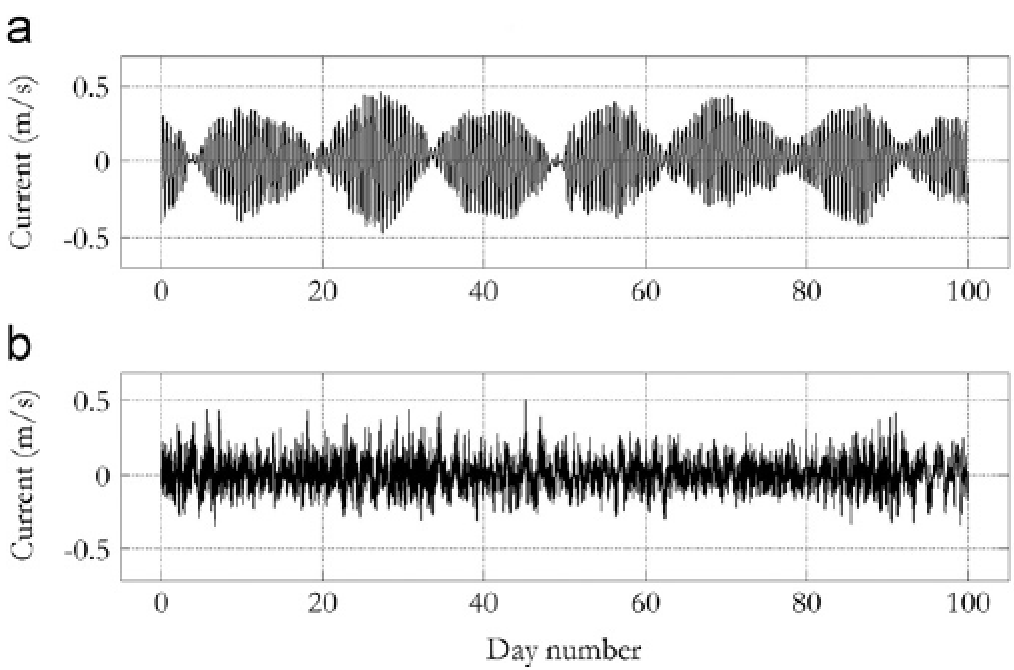
Multi-dimensional Example – Analysis (2)

- Tidal & Residuals separated by local harmonic analysis

$$c_{jk}(t) = a(t) + b_1 \sin\left(\left(24/T_1\right)t + \phi_1\right) + b_2 \sin\left(\left(24/T_2\right)t + \phi_2\right)$$

T_1 = 12.42 hrs (principal lunar - M_2)

T_2 = 12 hrs (principal solar - S_2)



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Multi-dimensional Example – Analysis (3)

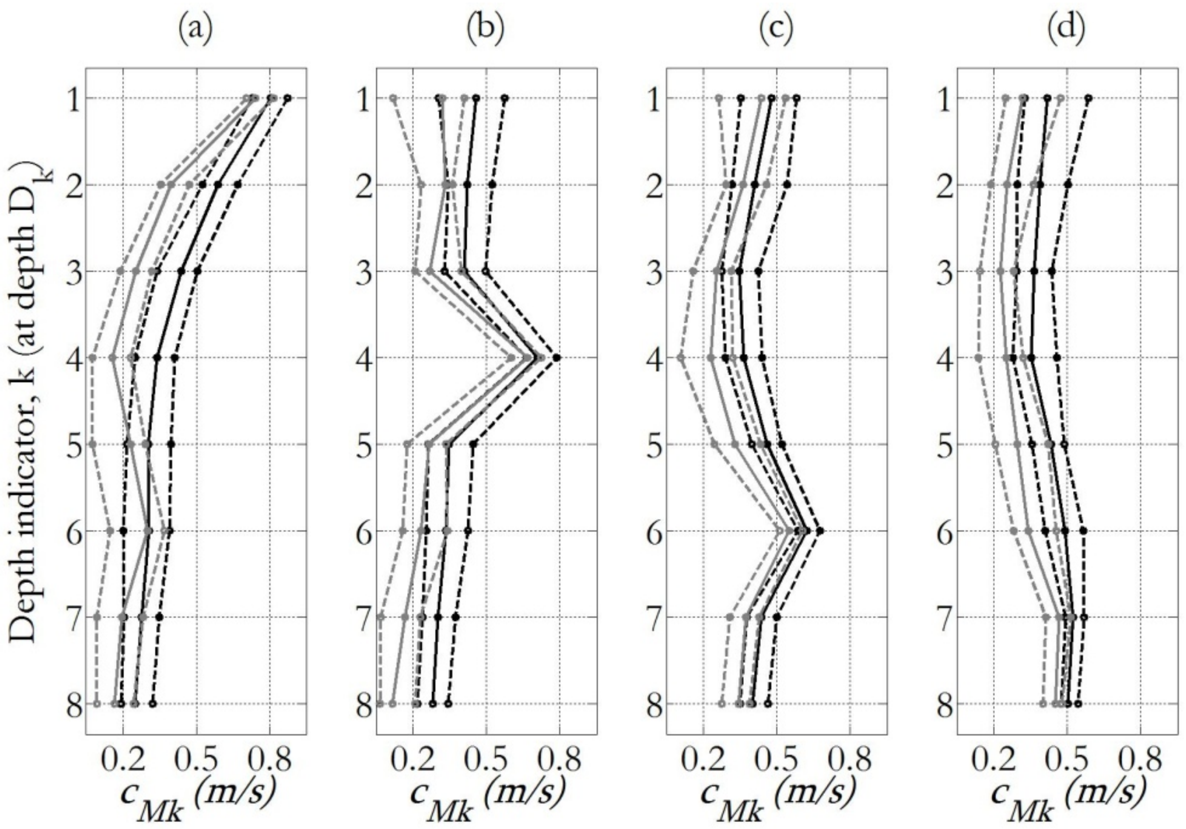
- Hourly maxima taken of tidal and residual components
  - Residual maxima for extreme value analysis
  - Tidal maxima for recombining with residual simulations
- Conditional extremes model applied to residual hourly extrema
  - Marginals fitted with Generalised Pareto
  - GPs transformed to Gumbel
  - Multi-dimensional conditional model fit

$$\left(\mathbf{Y}_{-k} \mid \mathbf{Y}_k = y_k \right) = \mathbf{a}_k y_k + y_k^{\mathbf{b}_k} Z_k$$

- Joint extreme distributions established through simulation
  - Tidal components are re-sampled with replacement
  - Sampled tidal components and residuals are added to provide hourly estimates of hourly maxima and minima along the major and minor axes.

Conditional Total Current – Major Axis

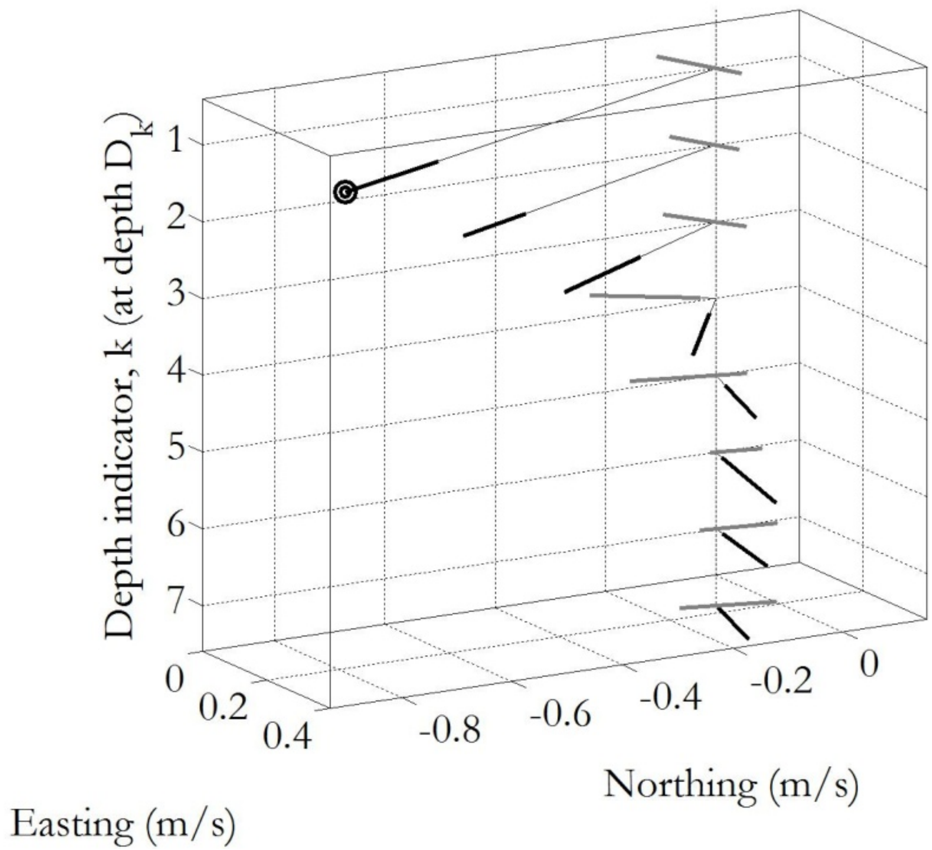
Median Monthly Maxima: black – conditional simulations + 25%, 75% Con. Lim.
grey – measurements



(a) c_{M1} (b) c_{M4} (c) c_{M6} (d) c_{M8}

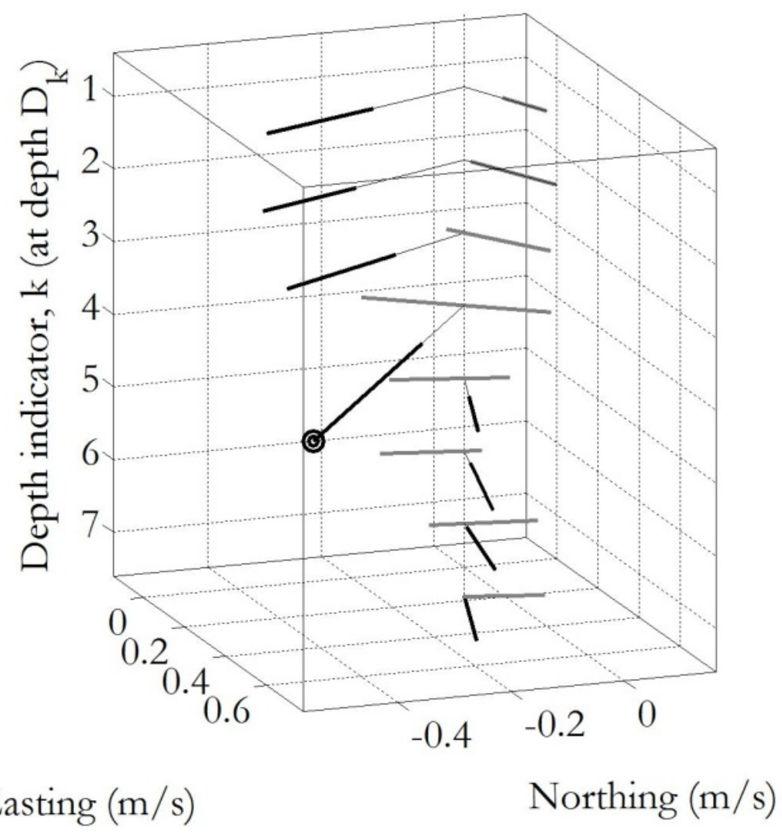
Conditional Total Current – Components

Median 10-yr maxima hourly extremes conditioned on c_{M1}



Conditional Total Current – Components

Median 10-yr maxima hourly extremes conditioned on c_{M4}



Conditional Extremes - Outline

- **Objective**: to model the joint distribution of extremes of X_1 and X_2 as a function of θ
- **Approach**:
  - Follow Heffernan & Tawn (2004)
  - Marginal Model X_1 and X_2 as a function of θ
    - Quantile Regression (QR) below threshold
    - Generalised Pareto (GP) above threshold
- Transform to standard Gumbel variates X_{G1} ,X_{G2}
- Model X_{G2} given large values of X_{G1} using extension to conditional extremes model incorporating covariate

$$Tp \mid Hs = h, \Theta = \theta \quad = \quad \alpha_\theta h + h^{\beta_\theta} \left(\mu_\theta + \sigma_\theta Z \right)$$

Conditional Extremes - Outline

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    - Generalised Pareto (GP) above threshold
- Transform to standard Gumbel variates X_{G1}, X_{G2}
- Model X_{G2} given large values of X_{G1} using extension to conditional extremes model incorporating covariate
- Simulate for long return periods
- Generate samples of joint extremes on Gumbel scale
- Transform to original scale

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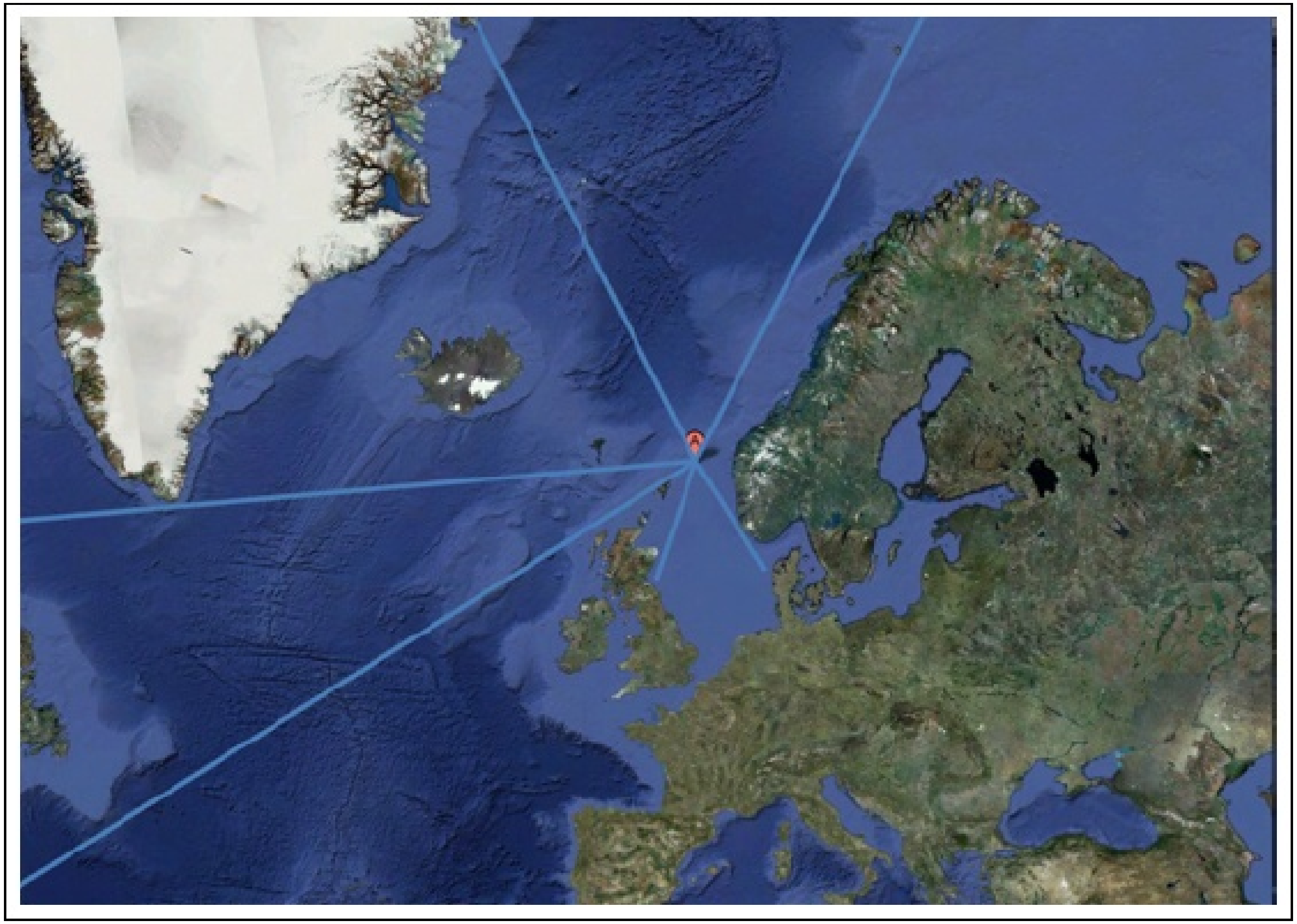
Conditional Extremes with covariates - Example



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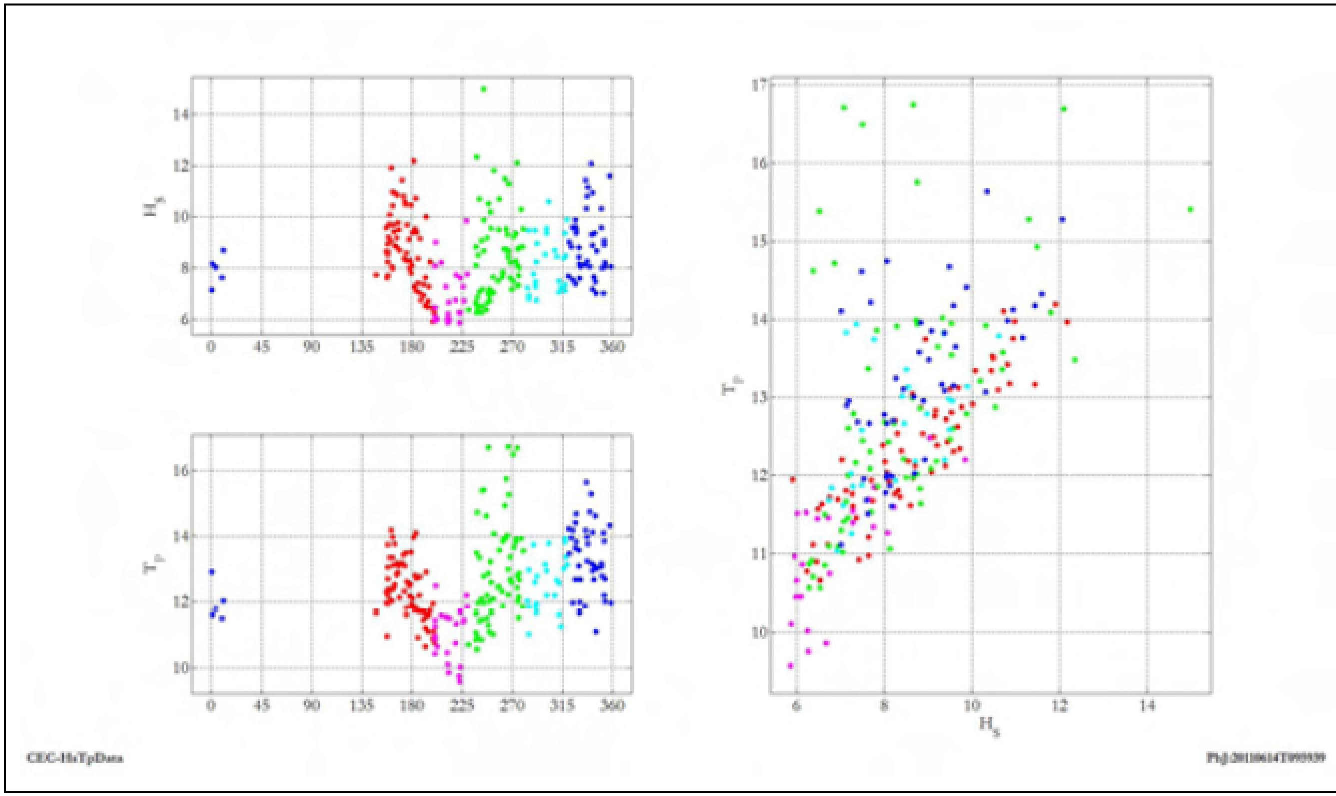
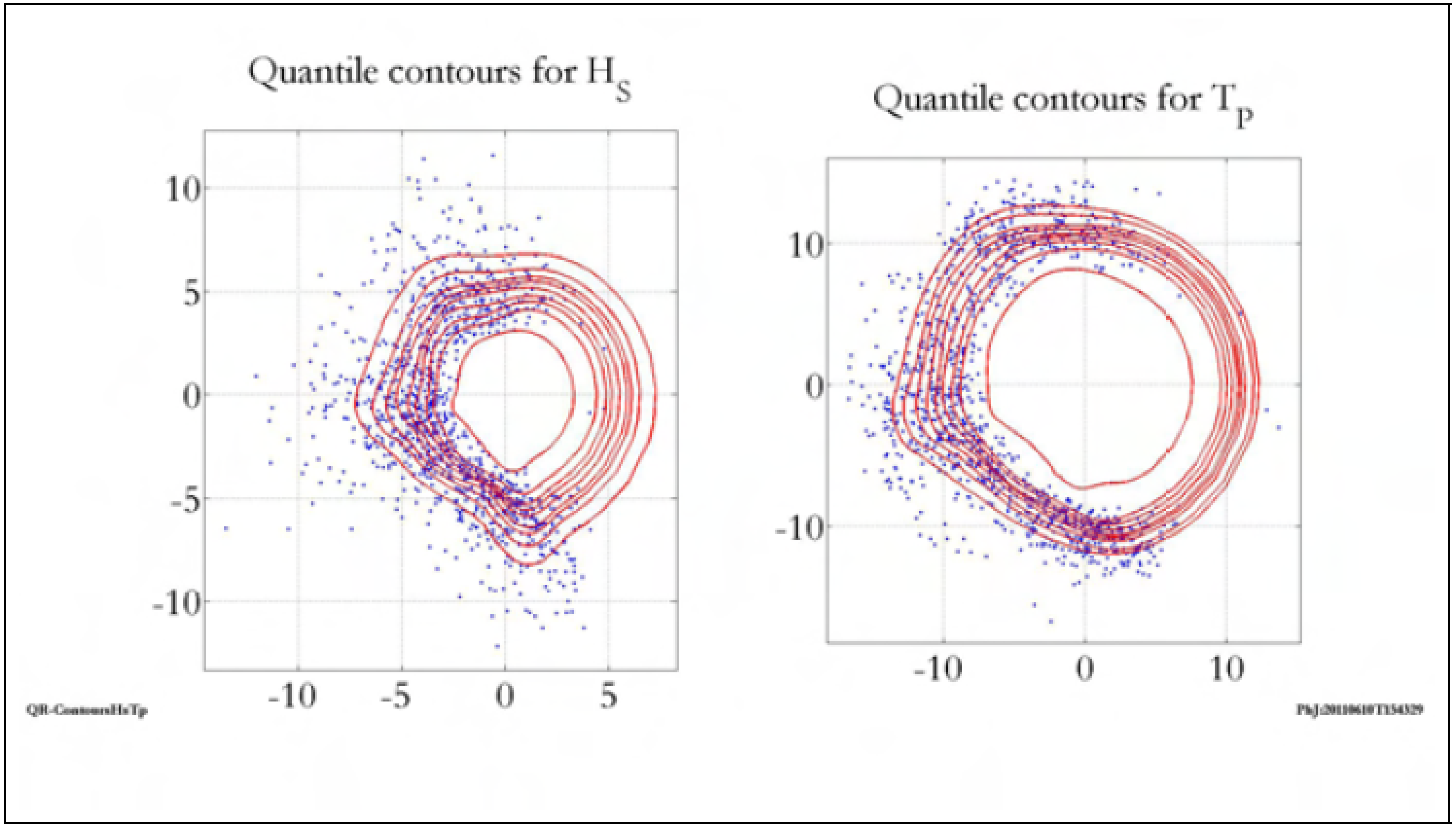
Conditional Extremes with covariates - Example



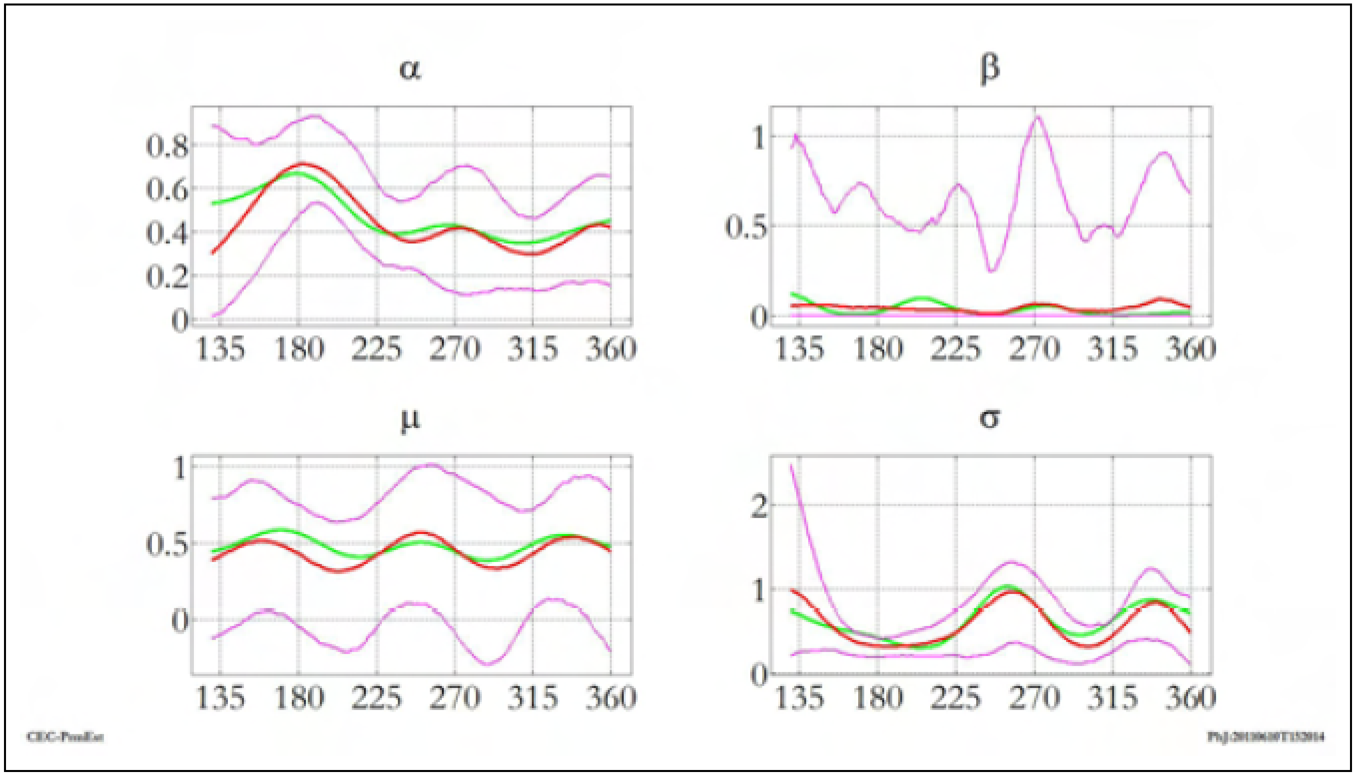
Spread of *Tp* **vs** *Hs* **different for different directions**

Conditional Extremes with covariates - Example



- **Transform directions to uniform prior using QR estimation**
- **Deciles to 80%**

Conditional Model - Parameter Estimates



$$Tp \mid Hs = h, \Theta = \theta \quad = \quad \alpha_\theta h + h^{\beta_\theta}\left(\mu_\theta + \sigma_\theta Z\right)$$

MLE in green; 1000 bootstrap resamples (median in red, 95% band in Magenta)

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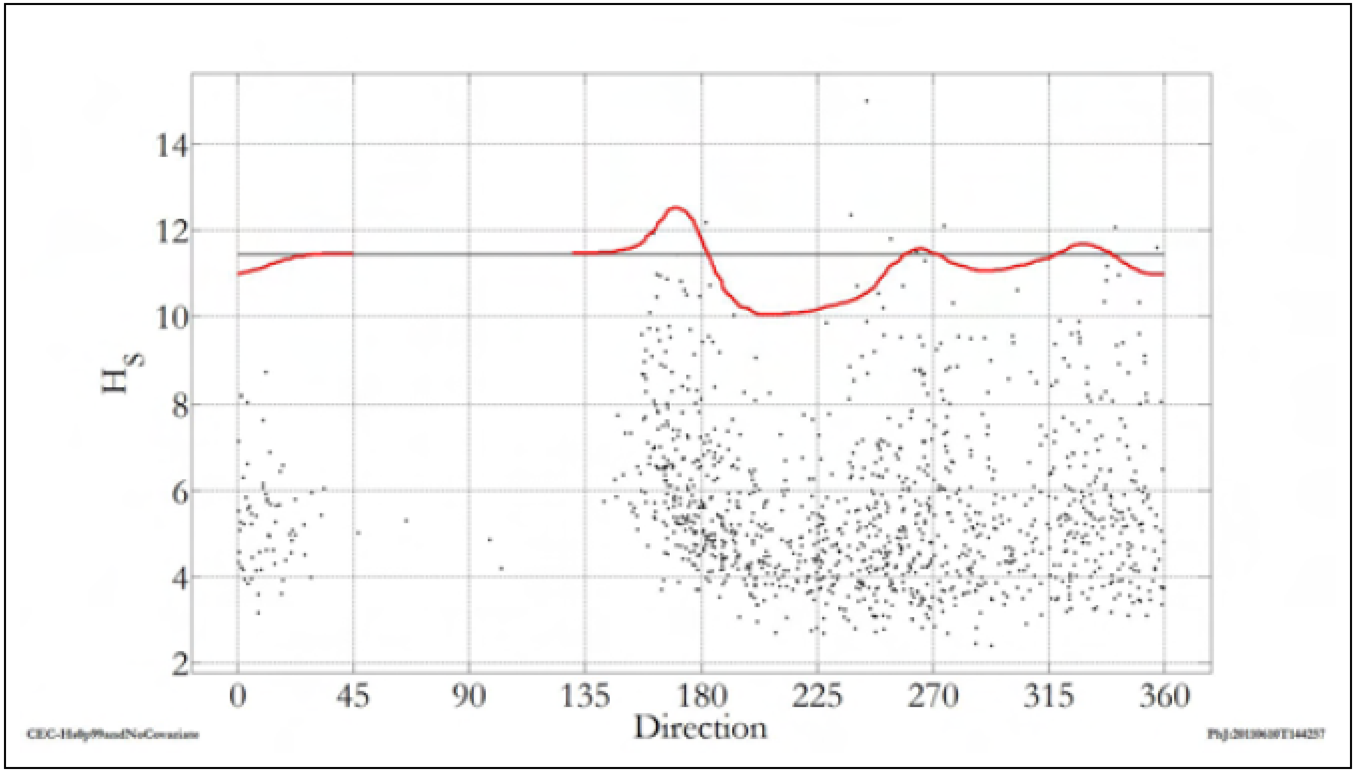
Conditioning variate Hs with tail probability = 0:01



Conditioning variate Hs with tail probability = 0:01

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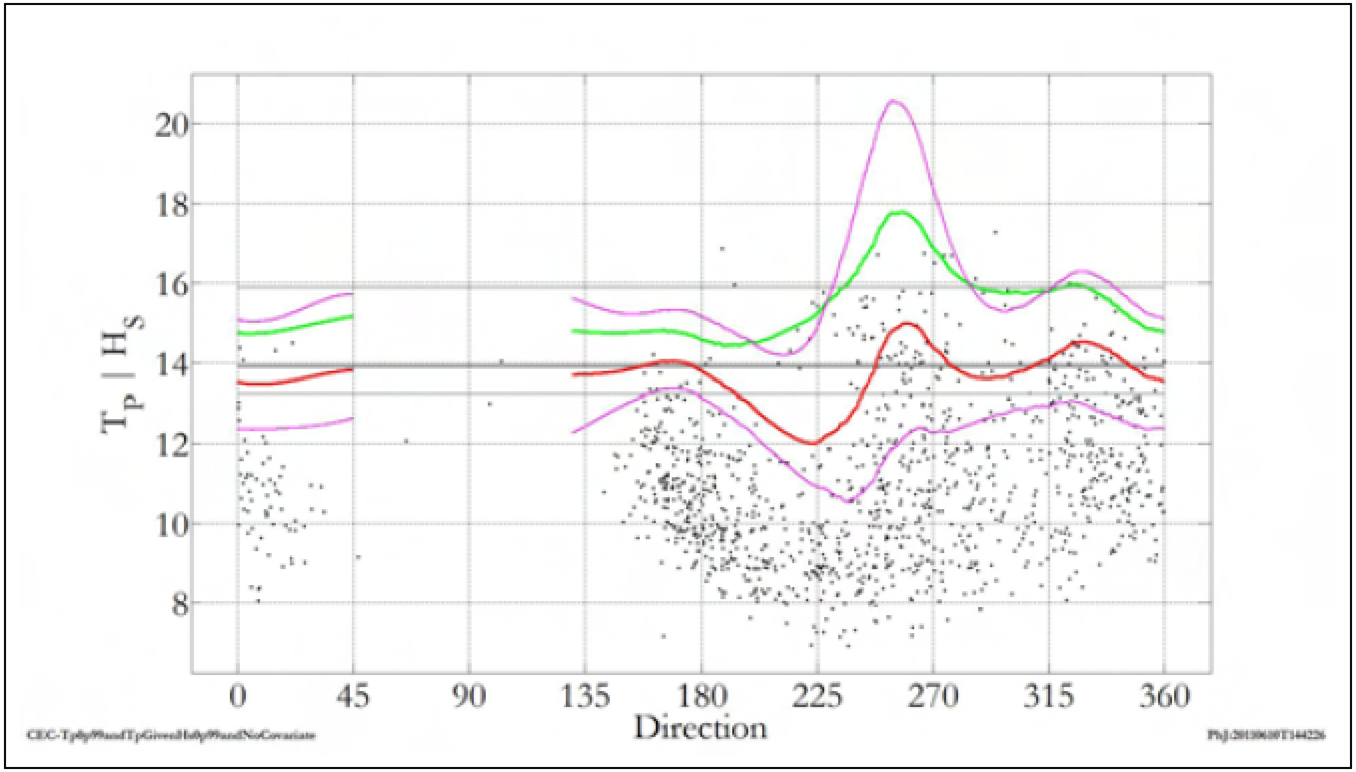
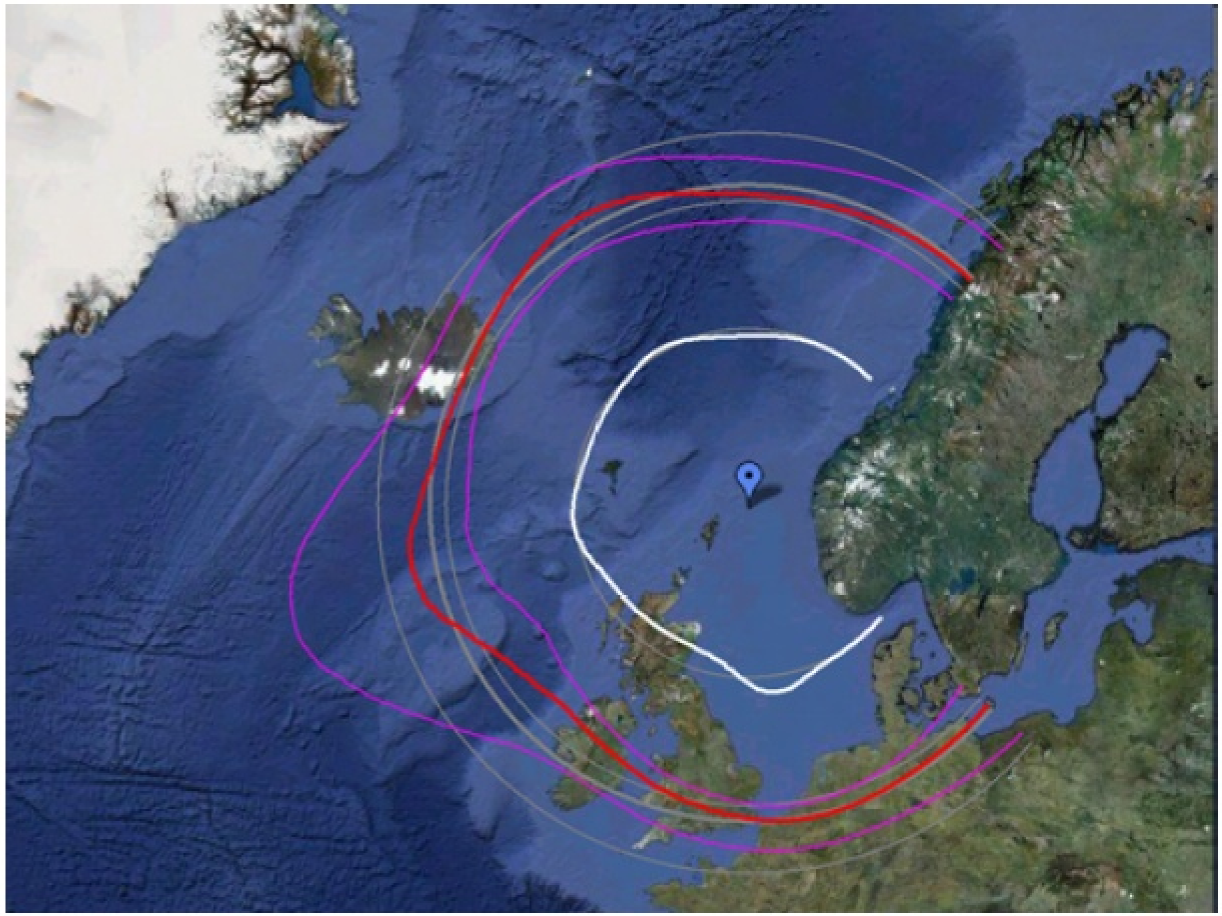
17-22 sept. 2012

Conditional Tp corresponding to Hs with tail probability = .01
Conditioning variate Hs with tail probability = 0:01



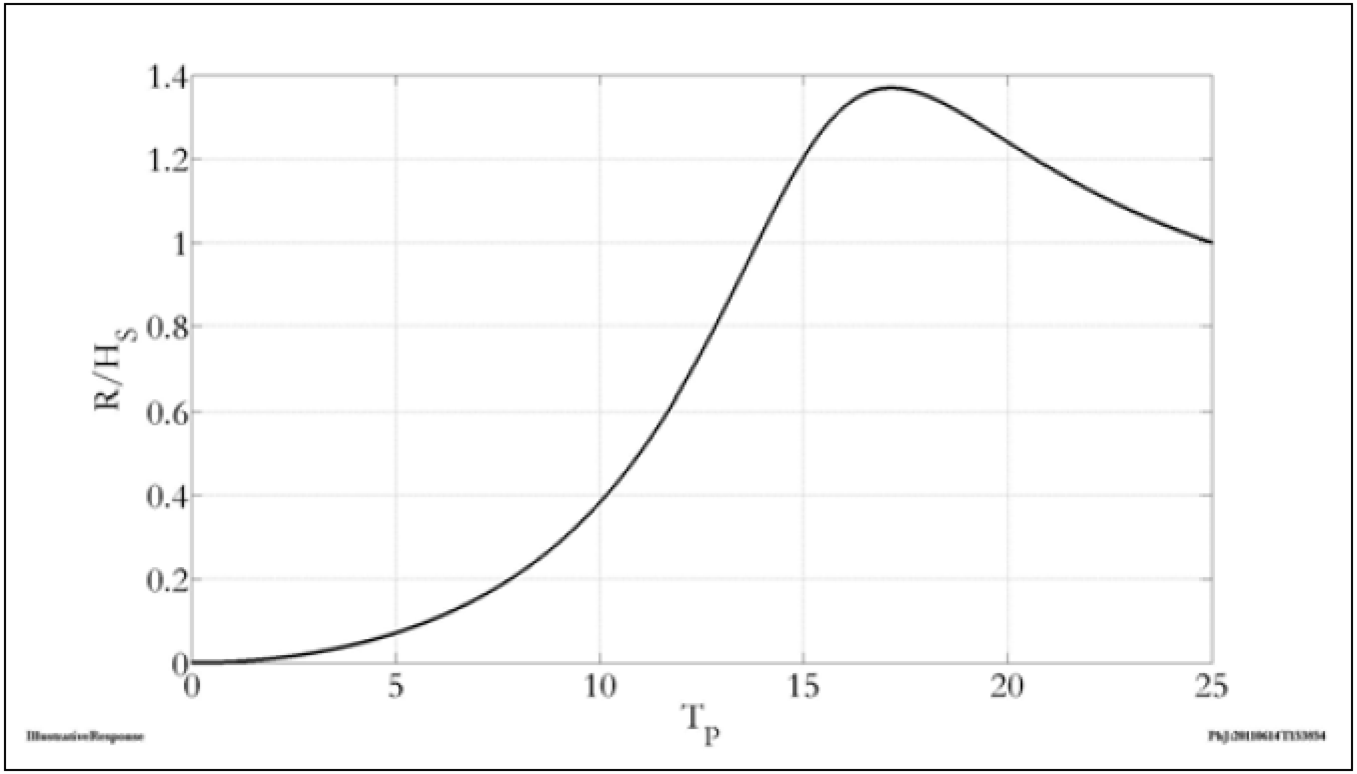
- **Tp with covariate (median (red), 95% band (magenta)), without (grey)**
- **Tp with exceedence probability = 0.01 shown in green**

Conditional Tp corresponding to Hs with tail probability = .01
Conditioning variate Hs with tail probability = 0:01



- **Tp (median (red), 95% band (magenta)), without (grey)**
- **Hs with exceedence probability = 0:01 shown in white**

Illustrative response transfer function



Characteristic of roll or heave response of floating structure:

$$\frac{R}{Hs} = \frac{1}{\sqrt{\left(1-\omega^2\right)+\left(k\omega\right)^2}}, \quad \omega = \frac{2\pi}{Tp}$$

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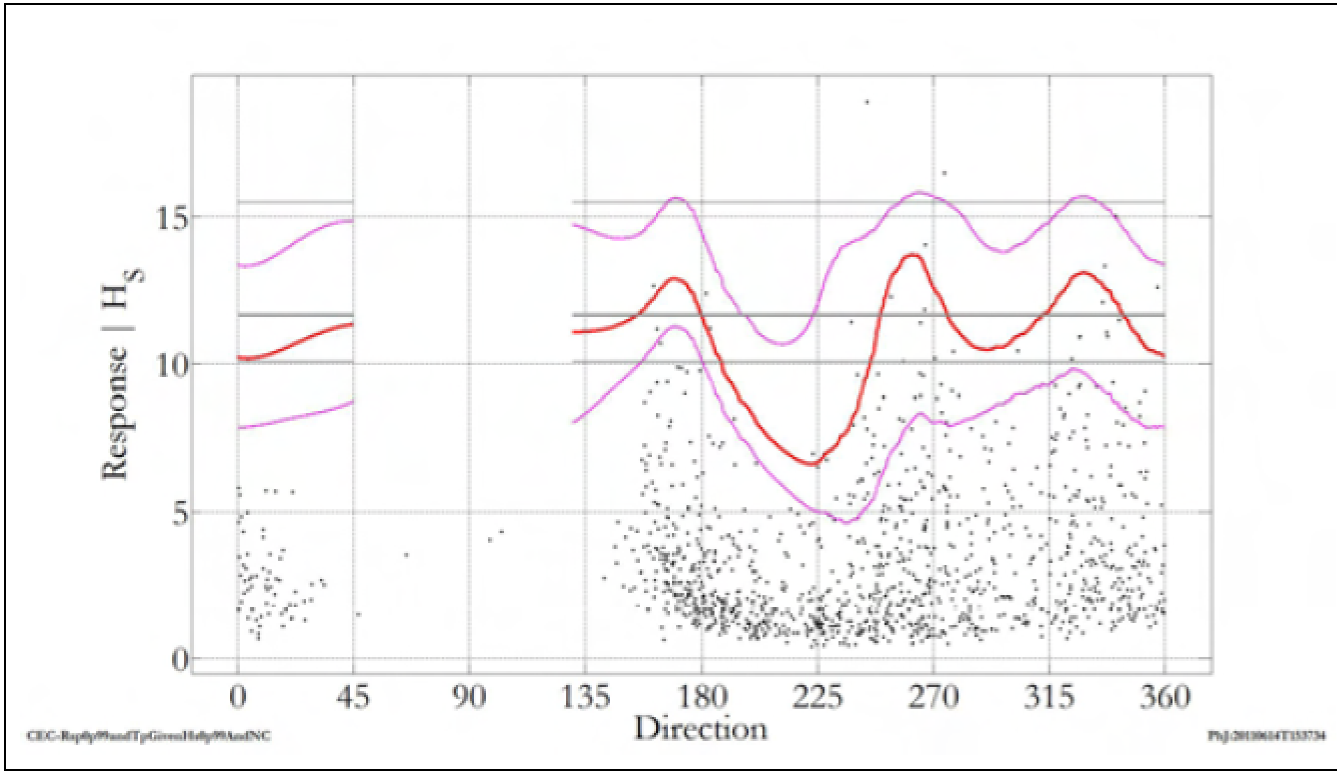
Conditional extreme response: with direction



- **Response with covariate effect (median in red, 95% limits in magenta) and without (grey) for Hs with tail probability = 0:01**

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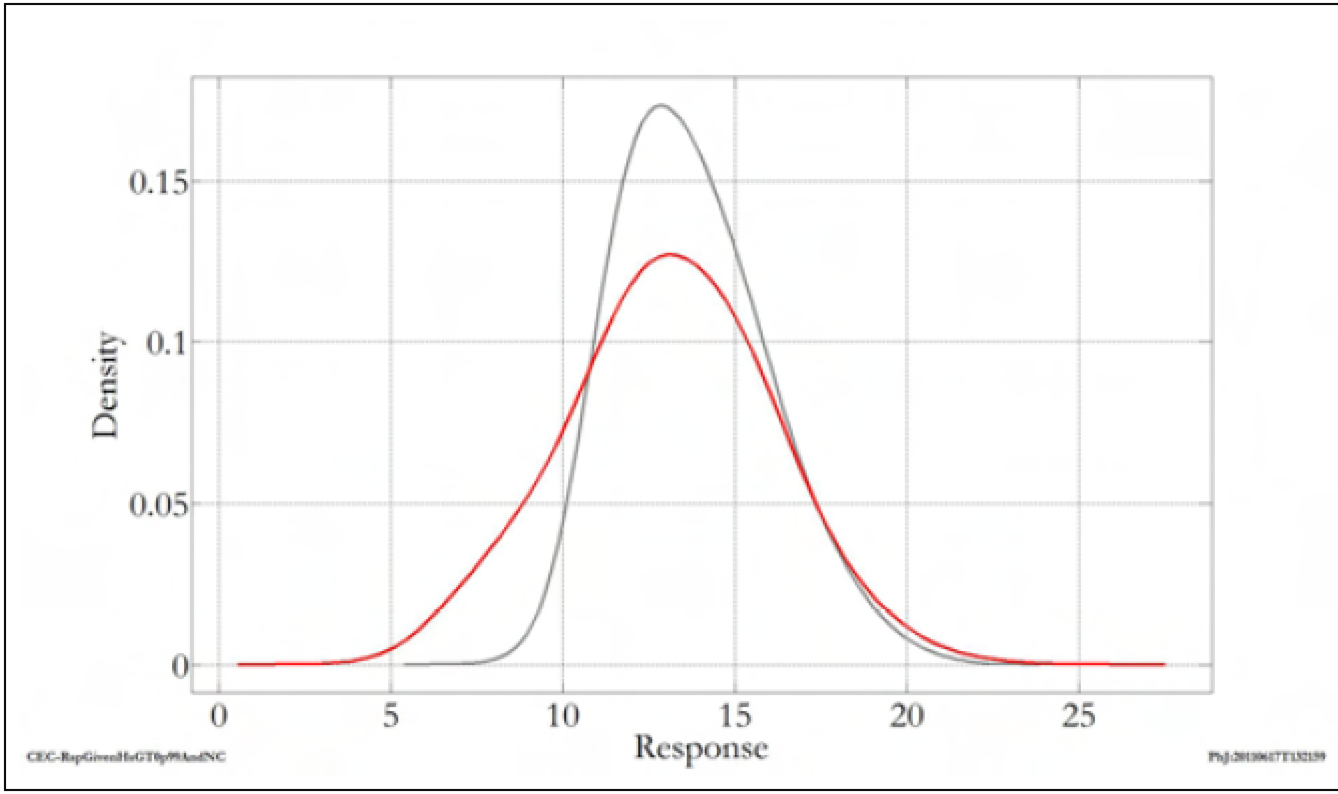
Conditional extreme response: kernel density estimate



- **Response density with covariate effect (red) and without (grey) for exceedences of Hs with tail probability = 0:01**

Summary

- **Methods exist to be more rigorous in modelling joint occurrences**
  - **dependence**
  - **uncertainty**
  - **design value estimates**
- **Conditional HT application easy and extends to multi-dimensional**
  - **generic model form for joint extremes**
  - **can incorporate covariates**