Wavelet Analysis for Sediment Transport Investigations

Craig Jones
Grace Chang

craigseaeng@gmail.com

Marine Science and Engineering Institute
Santa Cruz, CA, USA

« Time-series analysis in Marine science and applications for industry »
Conference in Logonna-Daoulas, France, 17-22 sept. 2012

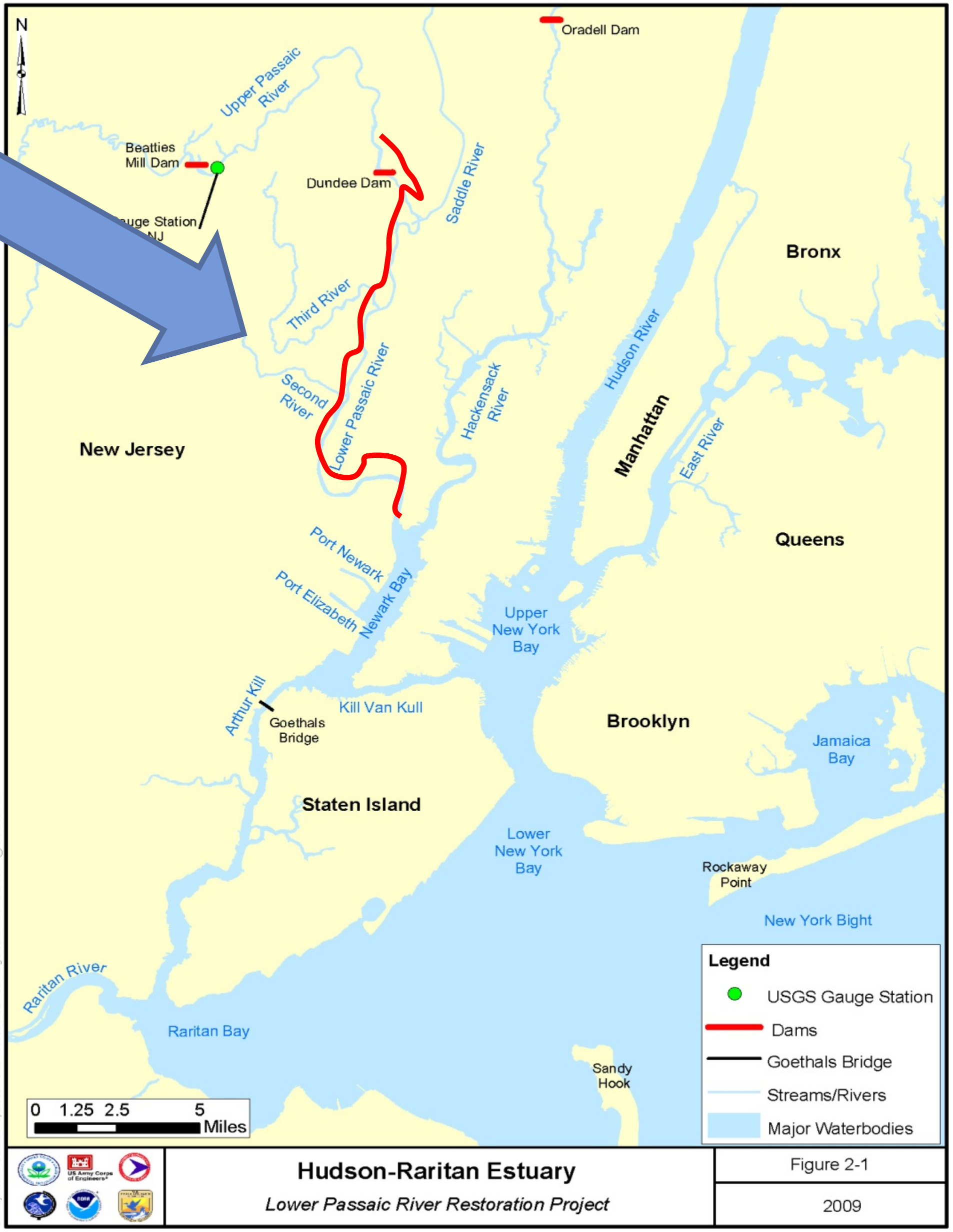
The purpose of computing is insight, not numbers.

Richard Hamming

Objectives

- **Background**
  - Passaic River
- **Observations**
- **Wavelets**
- **Results**
  - Continuous Wavelet Transforms
  - Cross Wavelet Transforms
  - Wavelet Coherence
- **Summary & Conclusions**

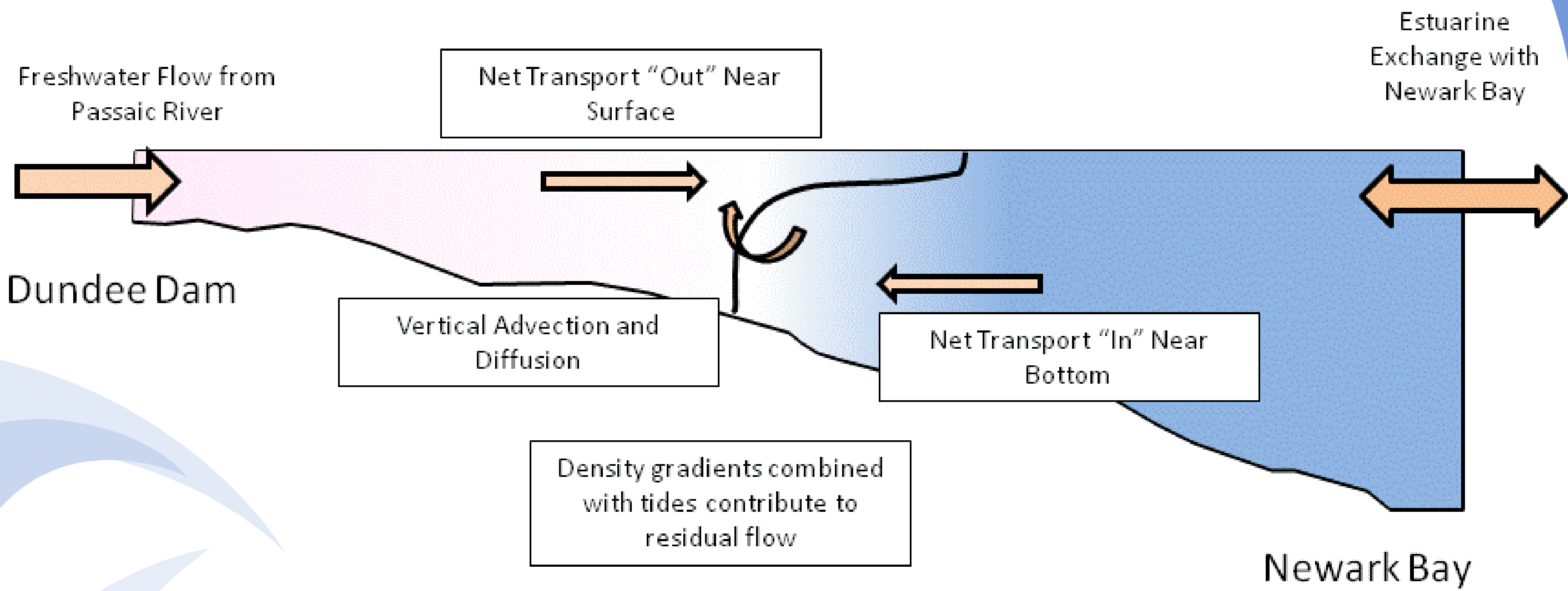
Raritan/Hudson Estuary and the Passaic River



Why do we care about sediment transport?

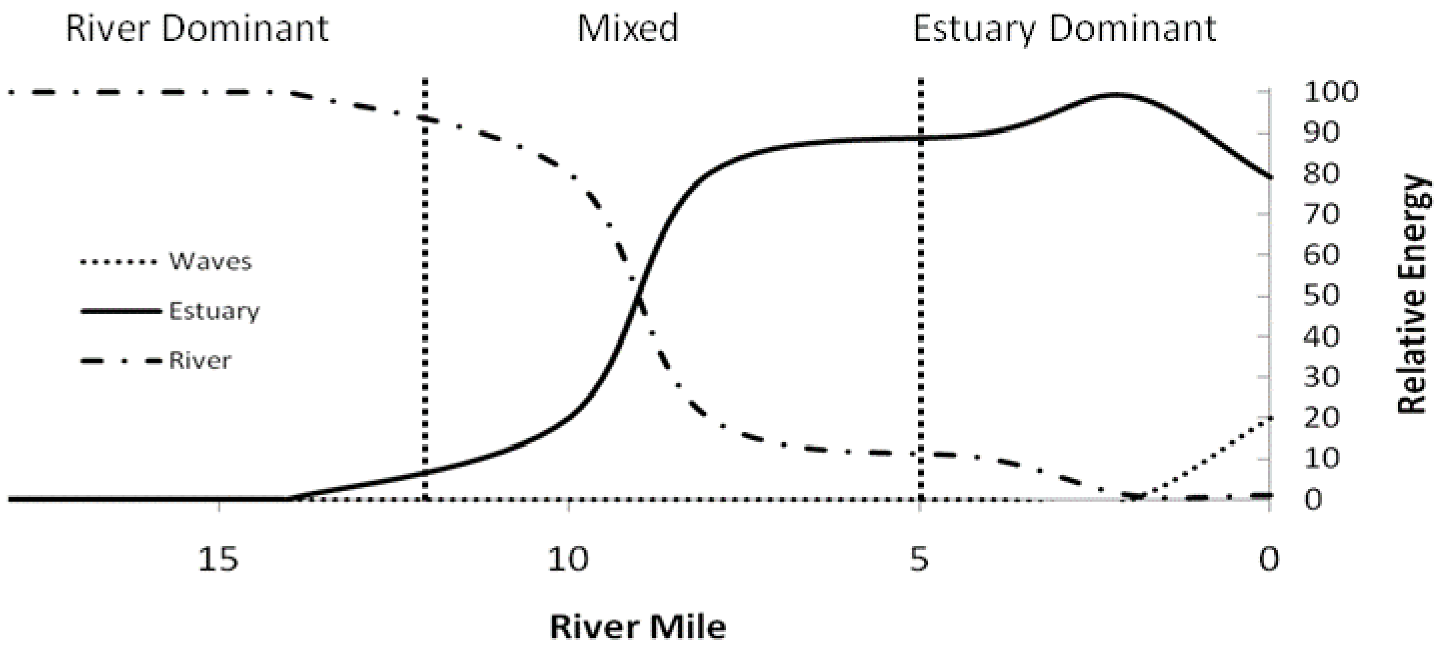
- The Lower Passaic River (LPR) below the Dundee Dam is contaminated with a range of persistent chemicals
- Many of these contaminants are hydrophobic and therefore strongly sorb to sediments in the system.
- Since the contaminants are generally strongly sorbed to the sediments, sediment transport is a key to understanding \environmental risk and \remedial selection

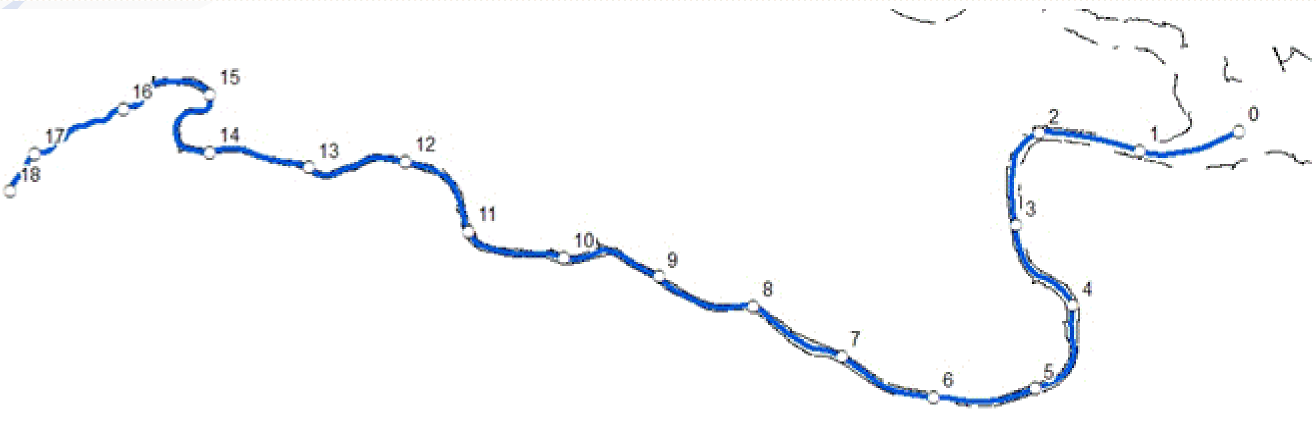
Estuarine Circulation



- Constantly adjusting balance between fresh river inflow, denser salt water in Newark Bay, and tidal mixing
- Net inflow on the bottom and net outflow at the surface is typical of apartially mixed estuarine circulation

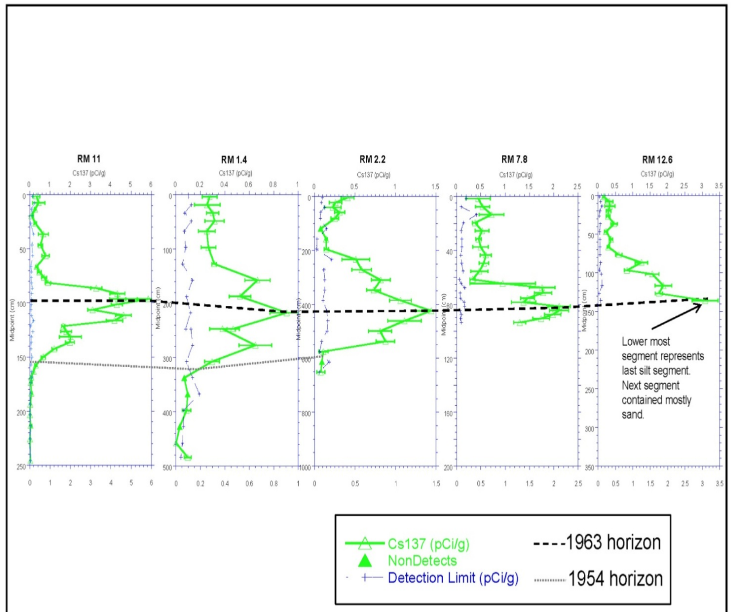
Process Balance





Long Term Behavior

- Bathymetric data and geochronology analysis show a net infill of sediments in the LPR since the significant dredging in the 20th century.

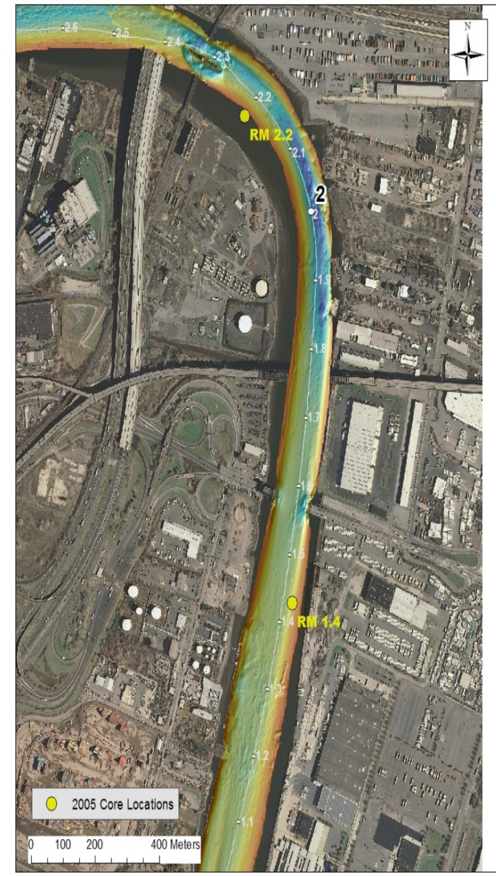


Cs-137 Downcore Profiles for 2005 High Resolution Cores

Figure 13-3

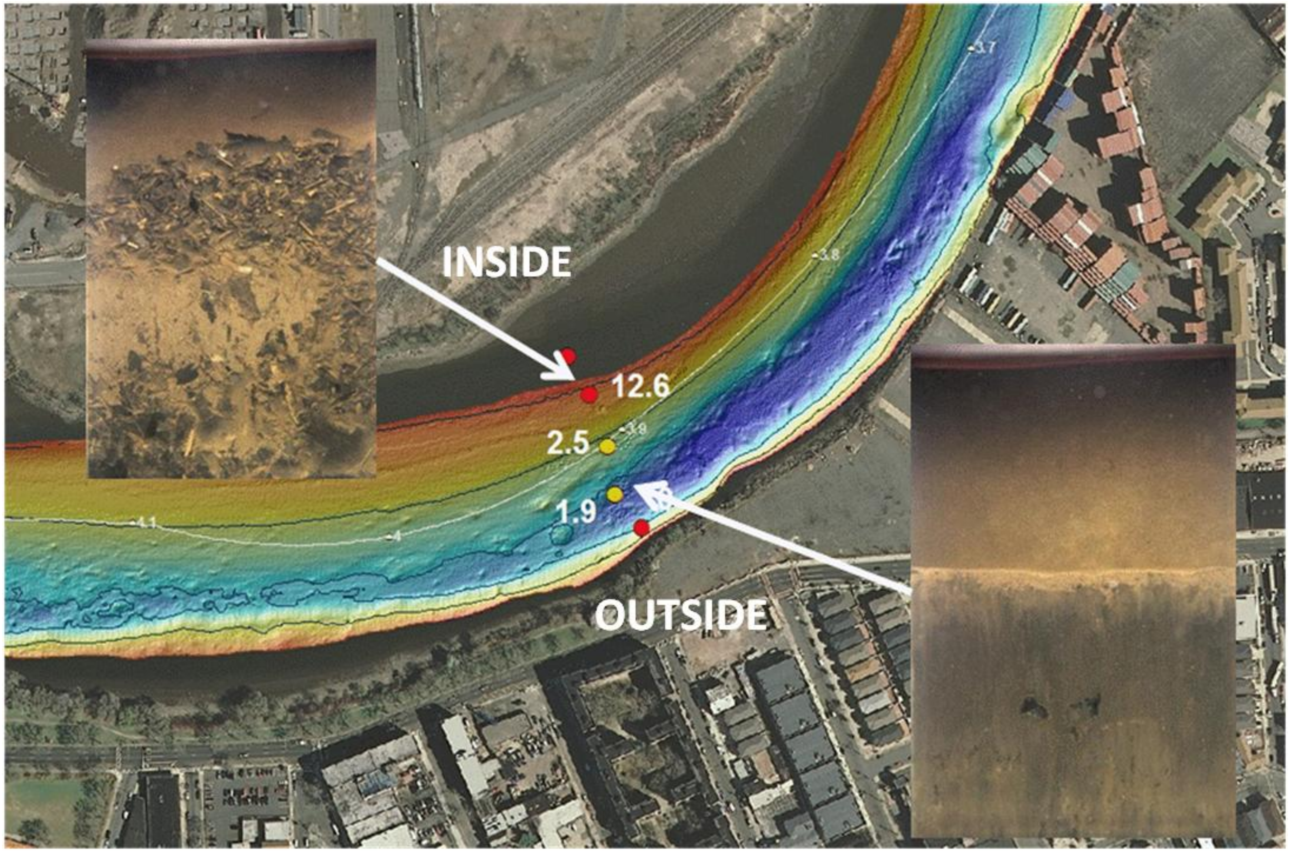
2009

Lower Passaic River Restoration Project



Sediment Bed Trends

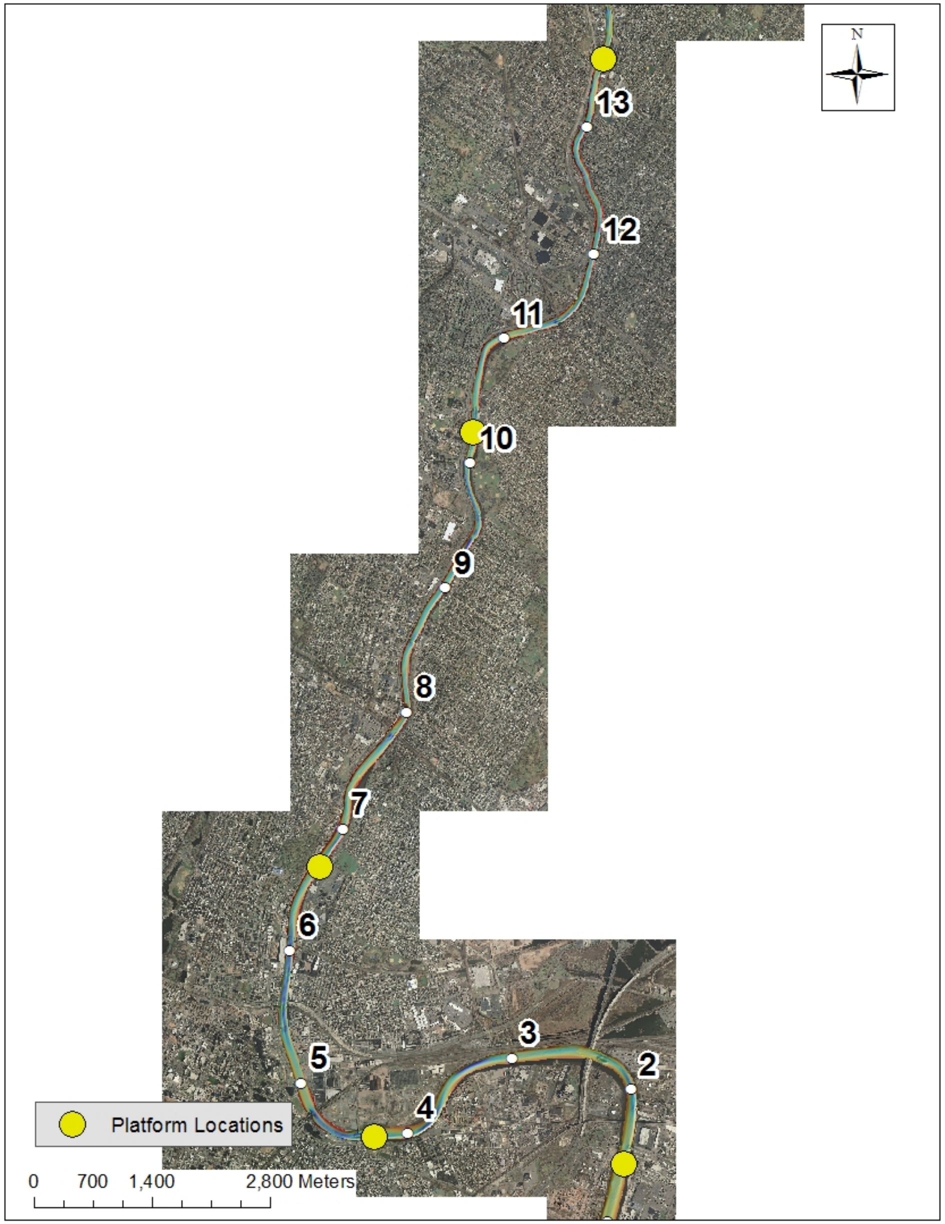
- Small scale heterogeneity at the bed follows morphologic features
- The sediment bed generally behaves consistent with standard river morphology



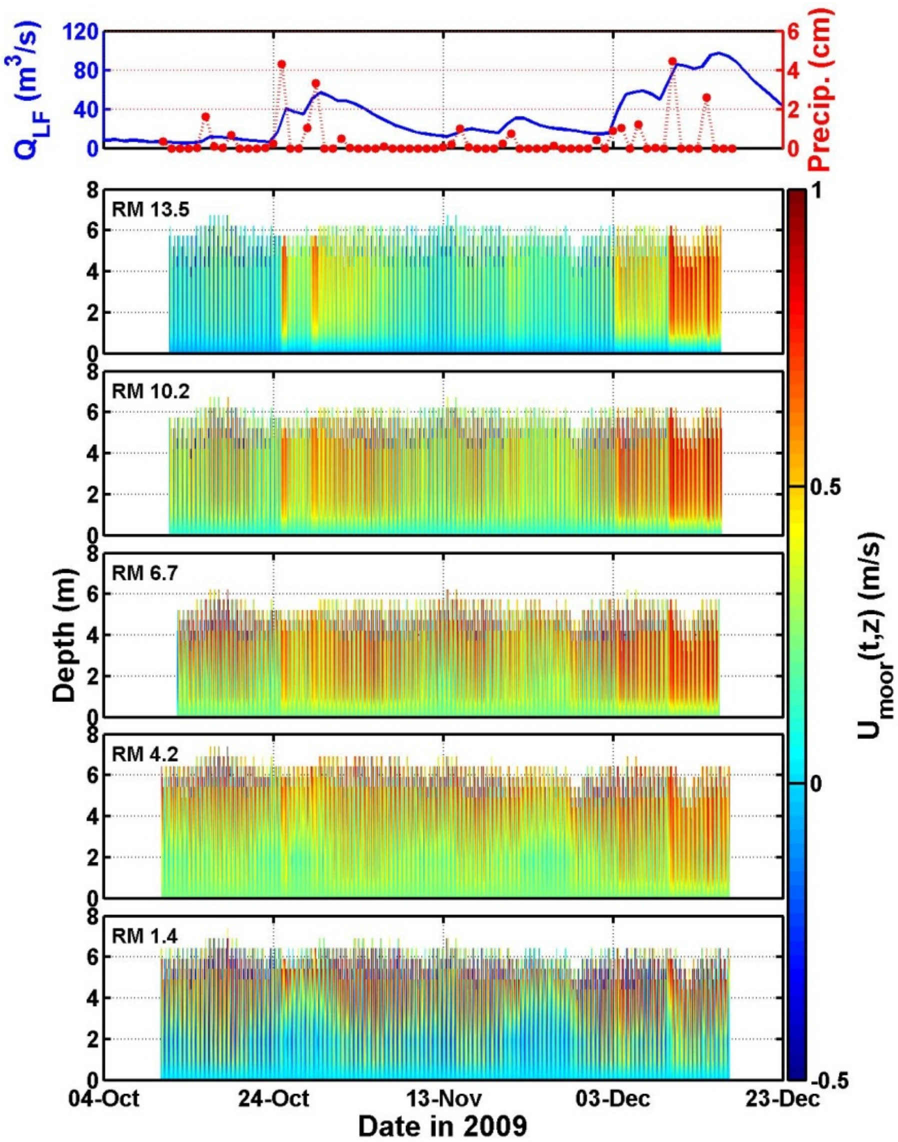
Water Column Measurements

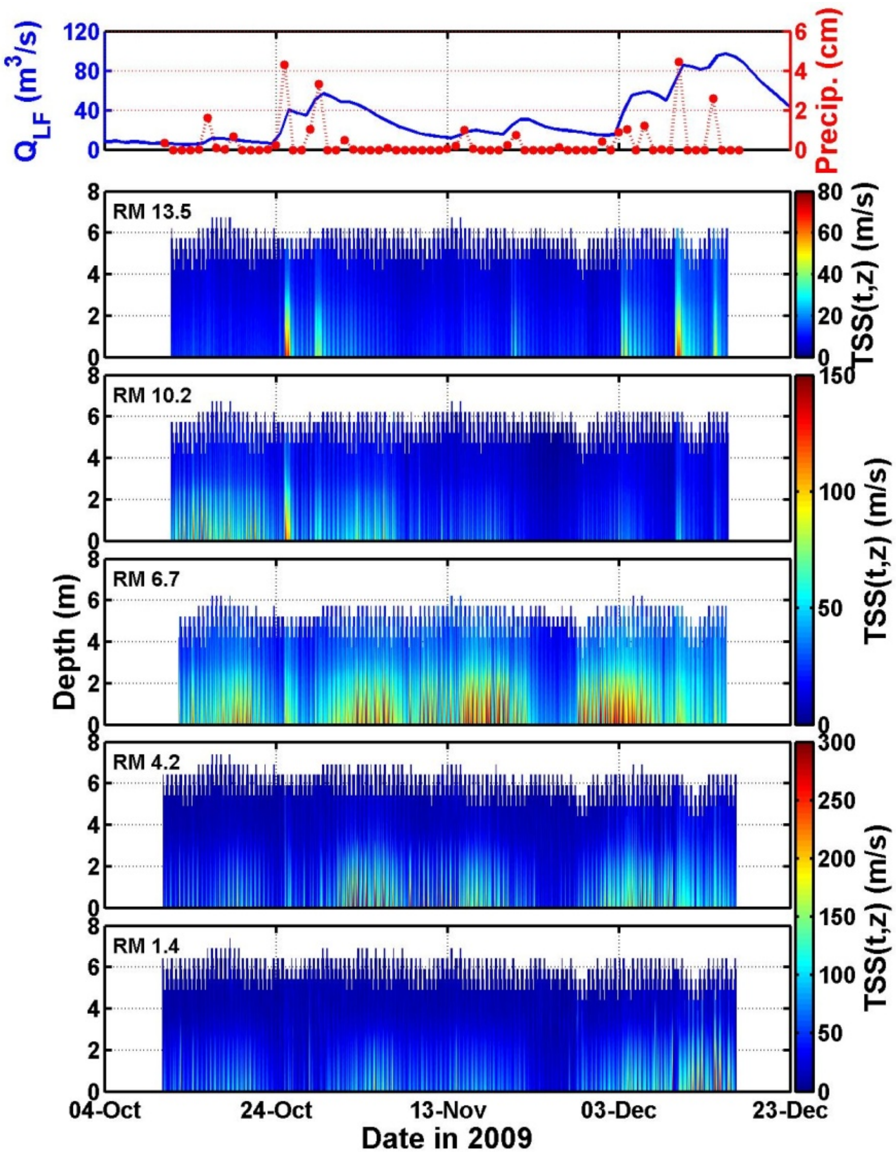
- 5 monitoring platforms were placed in the river from October through December

- The platforms measured currents, turbidity, temperature, and salinity

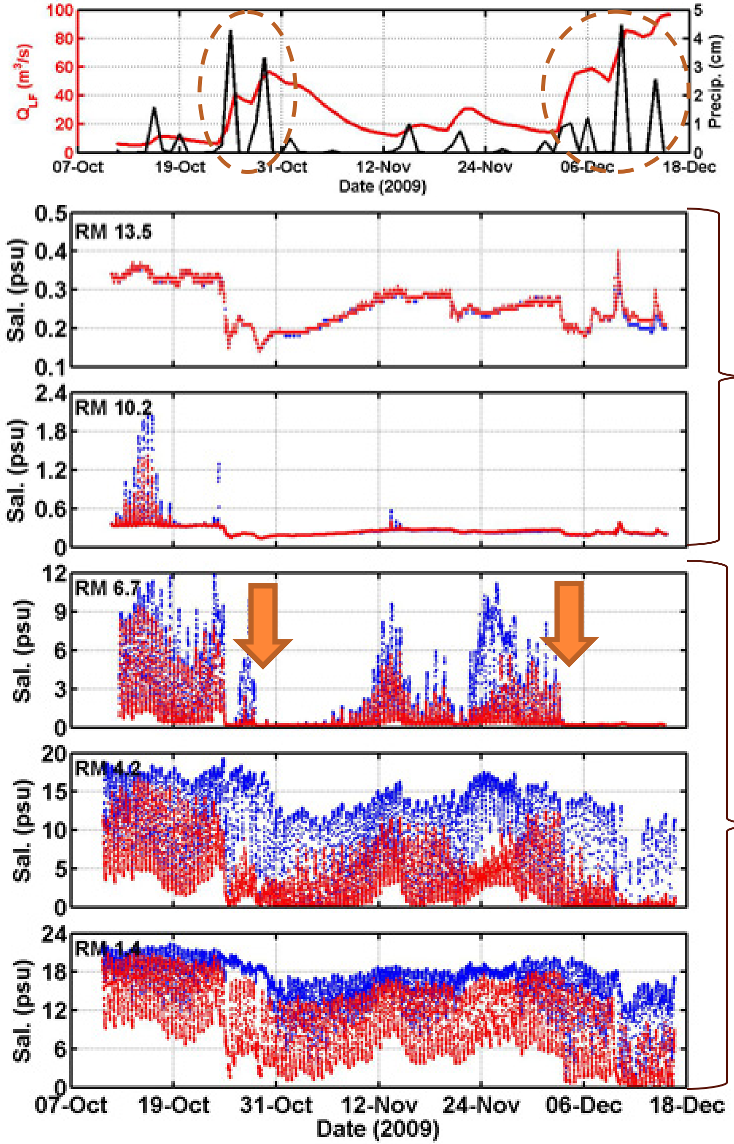


Water Column Measurements - Fall 2009





Observations



2 significant events

Strong gradient

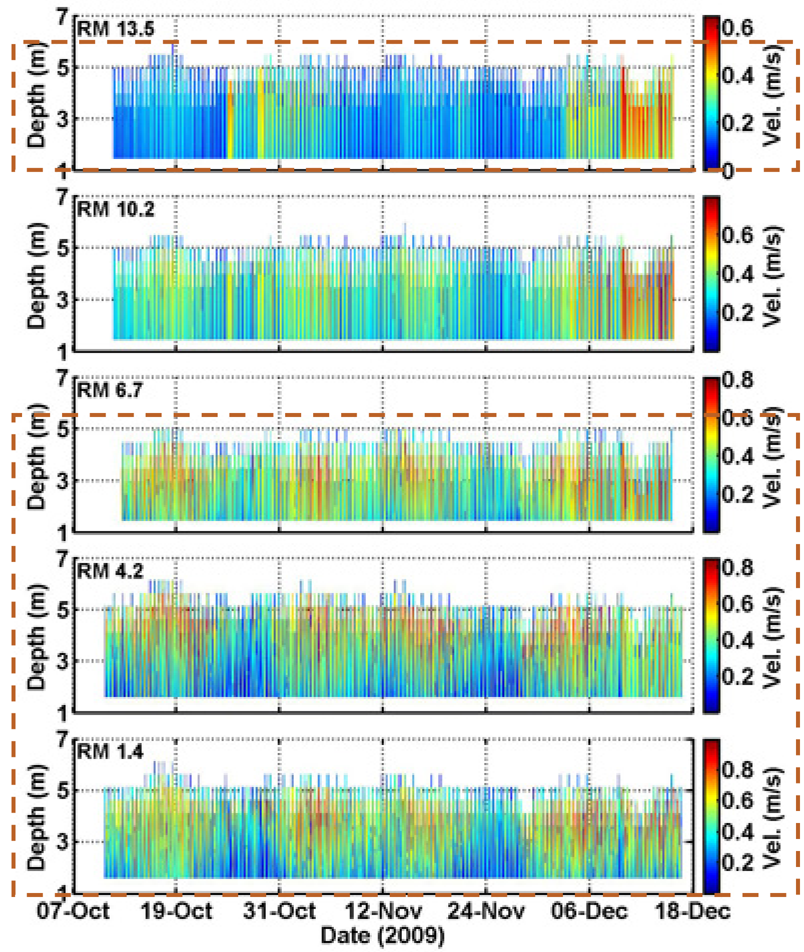
Freshwater flow dominates

Mixed

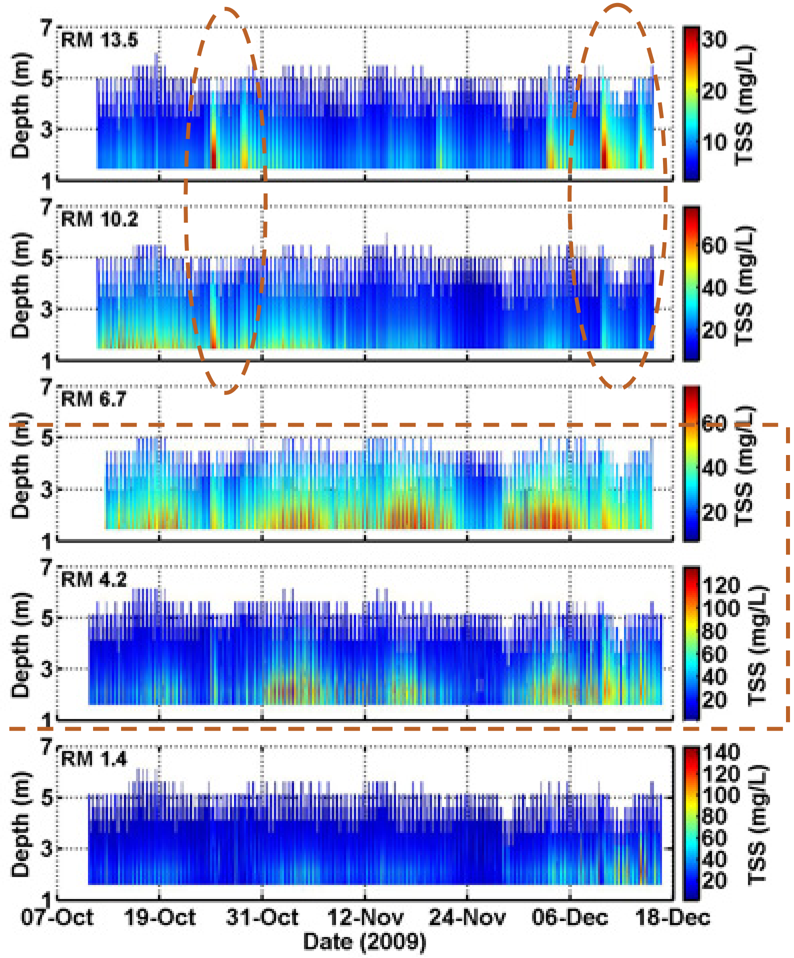
Stratified

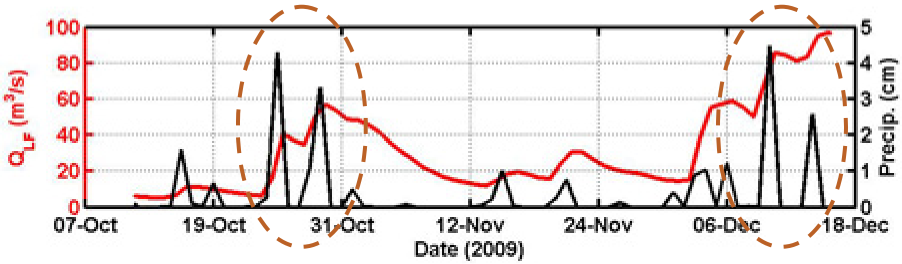
Tidal

Tidally dominated



Observations



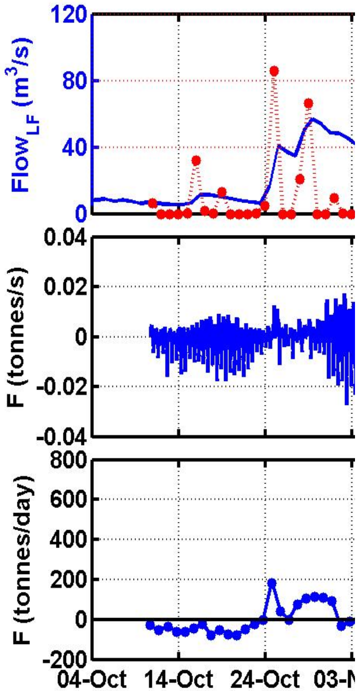


- TSS decreases with increasing RM
- Peaks in TSS and precipitation
- Spring-neap tidal oscillations

What are the underlying physical processes behind TSS variability in the LPR? When and where do they occur?

Platform Data Analysis

Dow



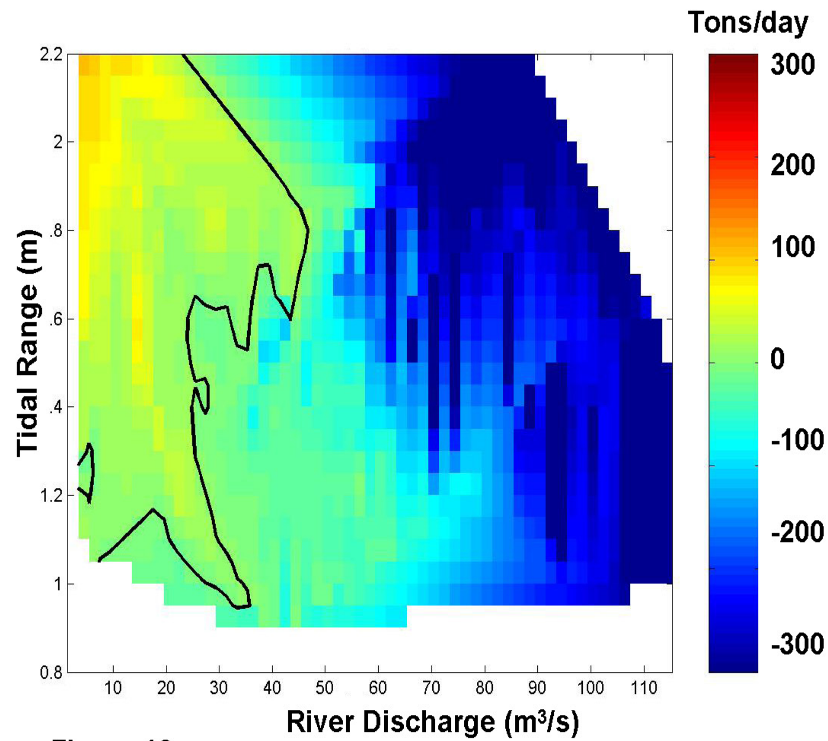
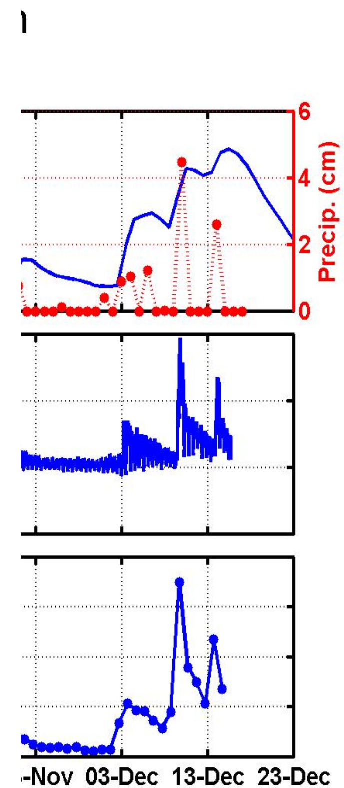


Figure 12

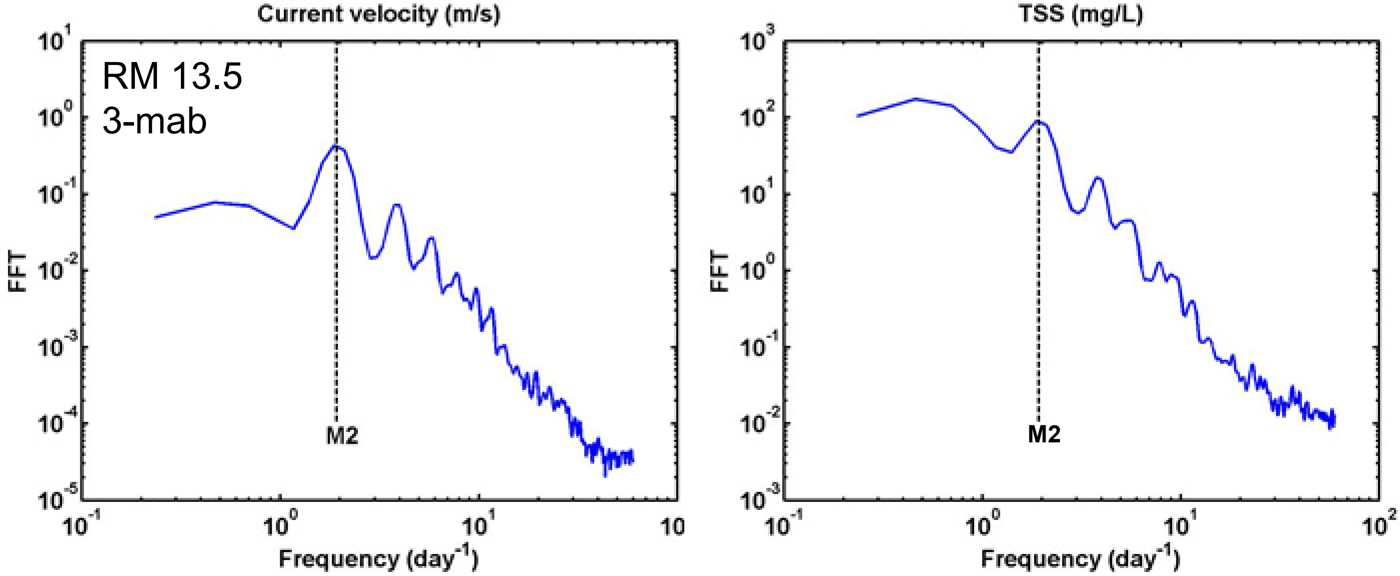


- Flux calculations at each platform show spatial and temporal trends in estuarine circulations (lower river) and riverine transport (upper river)

Time Series Analysis

- Fast Fourier Transforms (FFTs) are the traditional method of determining frequencies present in a time series signal.

FFT finds frequencies associated with, e.g., semi-diurnal tidal variability but they are not localized in time.



- Coherence finds common periodicities between two different time series but commonalities are not localized in time.

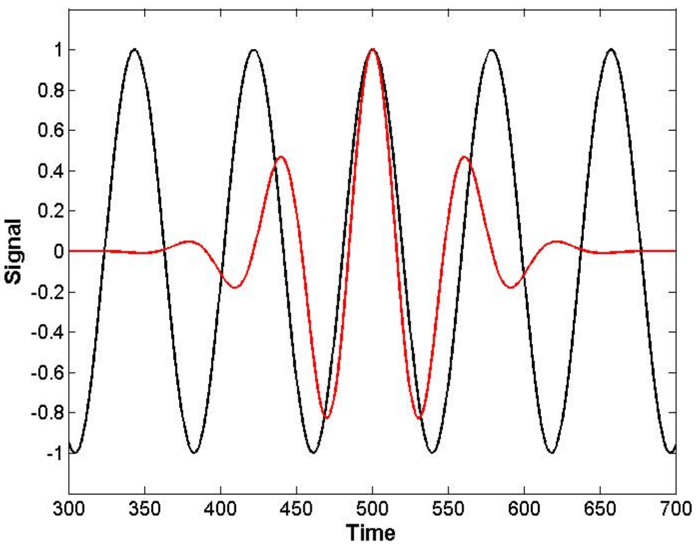
Time Series Analysis

- The Windowed Fourier Transform (WFT) also provides and analysis tool for extracting local frequency information
    - WFT is performed by essentially sliding a segment of defined length along the time series and performing short time Fourier transform
- Drawback of WFT
    - Imposes a response interval which causes problems with high and low frequency components
    - Method is highly dependent on window length
    - Generally not as effective for datasets with a larger range of frequencies

Wavelets

- Wavelet analysis to transform time series into time-frequency space

- Continuous wavelet transforms (CWTs): analyze localized, time series at multiple scales.



Morlet Wavelet

$$\psi_0(\eta) = \pi^{-1/4} e^{i\omega_0 \eta} e^{-\eta^2/2}$$

The wavelet can be translated to any point in the dataset (n) and scaled by a factor (s) which is directly related to frequency.

$$W_n(s) = \sum_{n'=0}^{N-1} x_{n'} \psi^* \left[\frac{(n'-n)\delta t}{s} \right]$$

Wavelet Transform

- Conduct Fourier transform on the wavelet

$$\hat{\psi}(s\omega_k) = \left(\frac{2\pi s}{\delta t}\right)^{\frac{1}{2}} \pi^{-1/4} exp[-(s\omega_k - \omega_0)^2/2]$$

- Conduct Fourier transform on the time series

- Choose a scale range (s), generally starting with 2dt to something less than ½ N

- Multiply normalized wavelet for each scale by transform of times series

$$W_n(s) = \sum_{k=0}^{N-1} \hat{x}_n \hat{\psi}^*(s\omega_k) e^{i\omega_k n\delta t}$$

Example

- A SST time series is transformed with wavelet analysis (Torrence and Compo 1998)
- A cone of influence is defined by the region of the wavelet transform where edge effects become significant
- Significance levels are outlined where confidence levels are 95% above background for the wavelet power spectrum

ENSO provides a substantive addition to the ENSO literature. In particular, the statistical significance testing allows greater confidence in the previous wavelet-based ENSO results of Wang and Wang (1996). The use of new datasets with longer time series permits a more robust classification of interdecadal changes in ENSO variance.

The first section describes the datasets used for the examples. Section 3 describes the method of wavelet analysis using discrete notation. This includes a discussion of the inherent limitations of the windowed Fourier transform (WFT), the definition of the wavelet transform, the choice of a wavelet basis function, edge effects due to finite-length time series, the relationship between wavelet scale and Fourier period, and time series reconstruction. Section 4 presents the theoretical wavelet spectra for both white-noise and red-noise processes. These theoretical spectra are compared to Monte Carlo results and are used to establish significance levels and confidence intervals for the wavelet power spectrum. Section 5 describes time or scale averaging to increase significance levels and confidence intervals. Section 6 describes other wavelet applications such as filtering, the power Hovmöller, cross-wavelet spectra, and wavelet coherence. The summary contains a step-by-step guide to wavelet analysis.

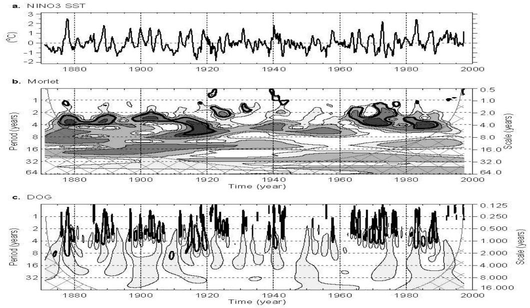


Fig. 1. (a) The Niño3 SST time series used for the wavelet analysis. (b) The local wavelet power spectrum of (a) using the Morlet wavelet, normalized by $1/\sigma^2$ ($\sigma^2 = 0.54°C^2$). The left axis is the Fourier period (in yr) corresponding to the wavelet scale on the right axis. The bottom axis is time (yr). The shaded contours are at normalized variances of 1, 2, 5, and 10. The thick contour encloses regions of greater than 95% confidence for a red-noise process with a lag-1 coefficient of 0.72. Cross-hatched regions on either end indicate the "cone of influence," where edge effects become important. (c) Same as (b) but using the real-valued Mexican hat wavelet (derivative of a Gaussian; DOG $m = 2$). The shaded contour is at normalized variance of 2.0.

2. Data

Several time series will be used for examples of wavelet analysis. These include the Niño3 sea surface temperature (SST) used as a measure of the amplitude of the El Niño–Southern Oscillation (ENSO). The Niño3 SST index is defined as the seasonal SST averaged over the central Pacific (5°S–5°N, 90°–150°W). Data for 1871–1996 are from an area average of the U.K. Meteorological Office GISST2.3 (Rayner et al. 1996), while data for January–June 1997 are from the Climate Prediction Center (CPC) optimally interpolated Niño3 SST index (courtesy of D. Garrett at CPC, NOAA). The seasonal means for the entire record have been removed to define an anomaly time series. The Niño3 SST is shown in the top plot of Fig. 1a.

Gridded sea level pressure (SLP) data is from the UKMO/CSIRO historical GMSLP2.1f (courtesy of D. Parker and T. Basnett, Hadley Centre for Climate Prediction and Research, UKMO). The data is on a 5° global grid, with monthly resolution from January 1871 to December 1994. Anomaly time series have been constructed by removing the first three harmonics of the annual cycle (periods of 365.25, 182.625, and 121.75 days) using a least-squares fit.

The Southern Oscillation index is derived from the GMSLP2.1f and is defined as the seasonally averaged pressure difference between the eastern Pacific (20°S, 150°W) and the western Pacific (10°S, 130°E).

Application to the Passaic River

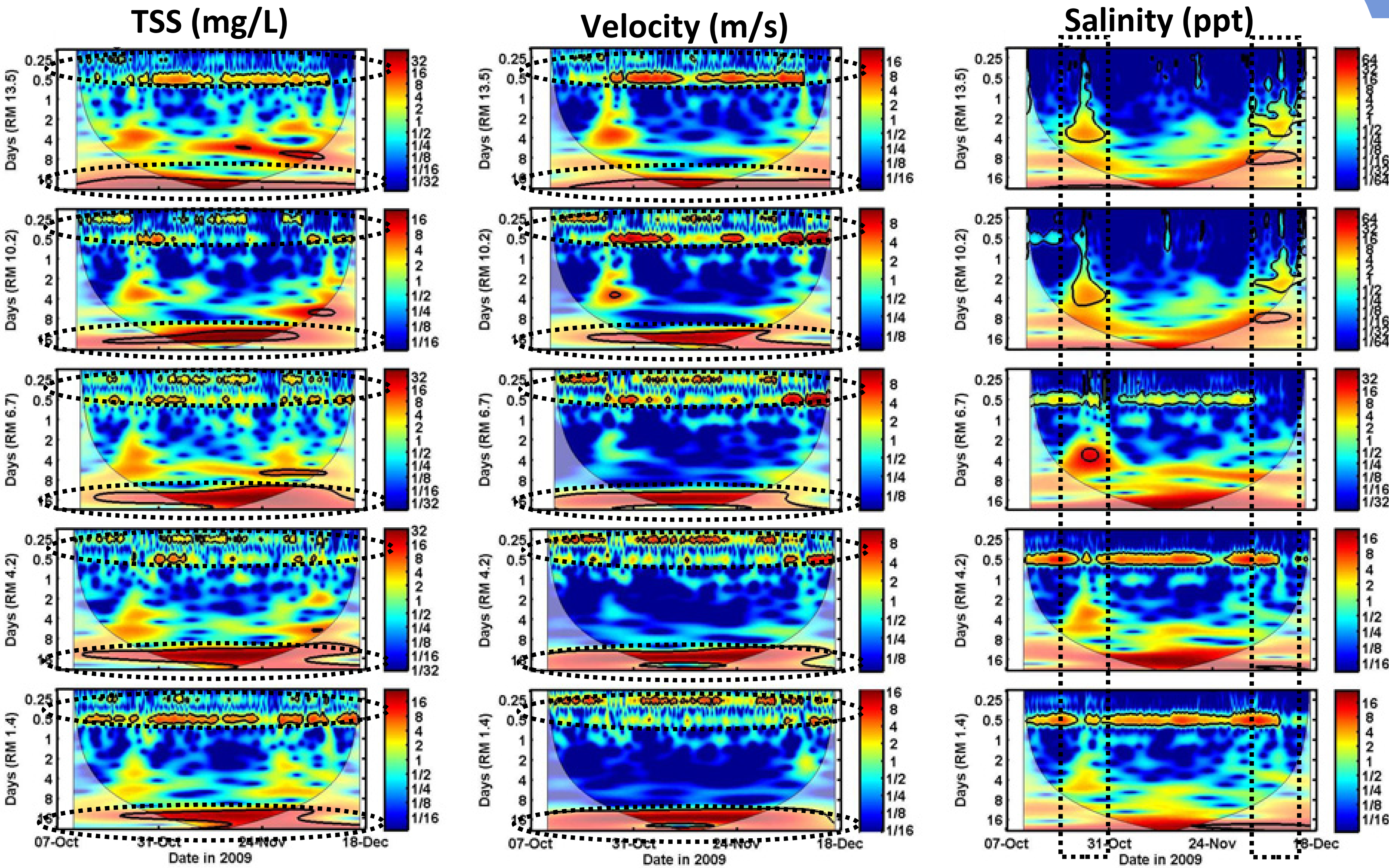
- Apply a wavelet to time series

- The result is a collection of time-frequency representations

A Wavelet analysis performed on velocity, salinity, and precipitation time series data collected along the LPR

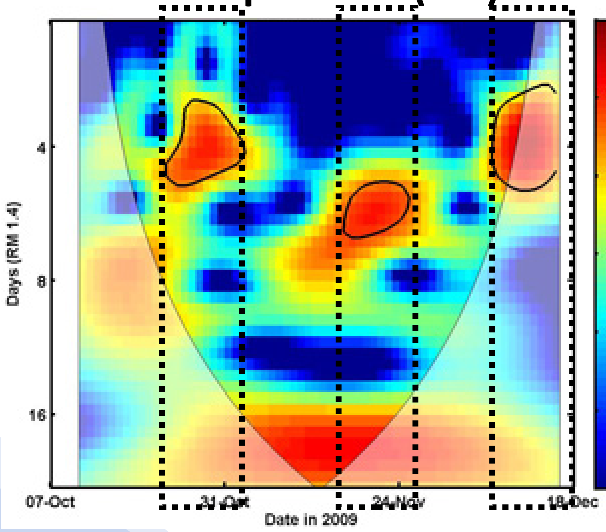
What is the spatial extent of tidally induced sediment resuspension and transport versus high sediment loads during river outflow due to precipitation events?

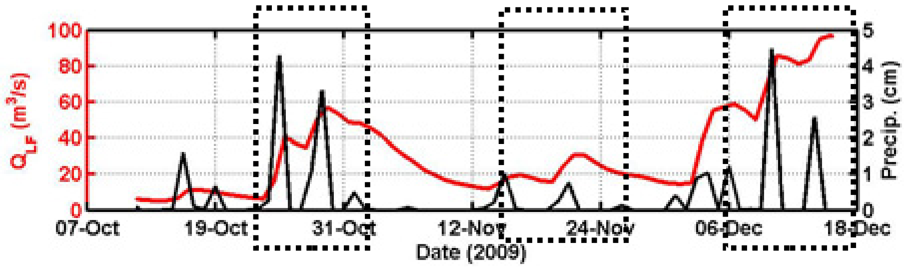
Results - CWTs



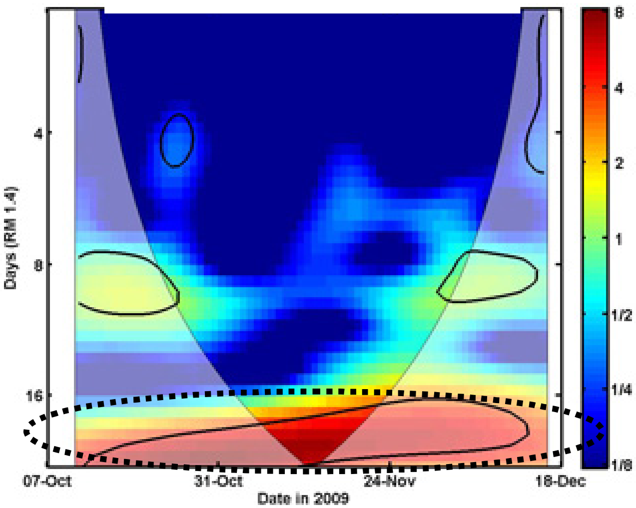
Results - CWTs

Precipitation (cm)

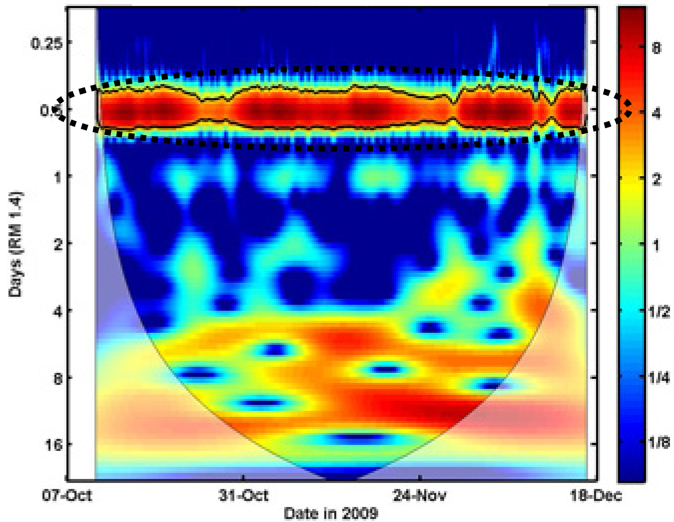




Q_{LF}(m^3/s)

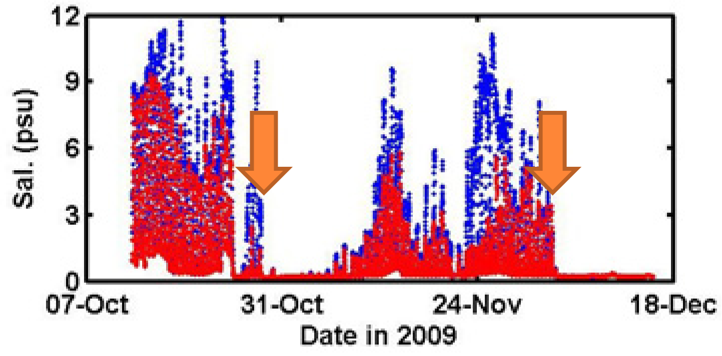


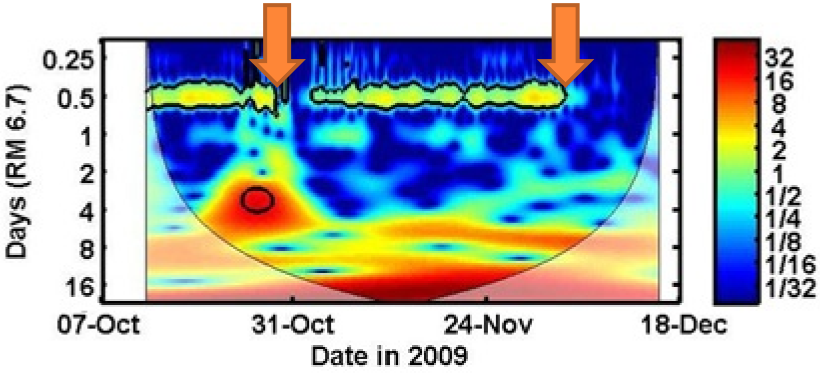
Depth (m)



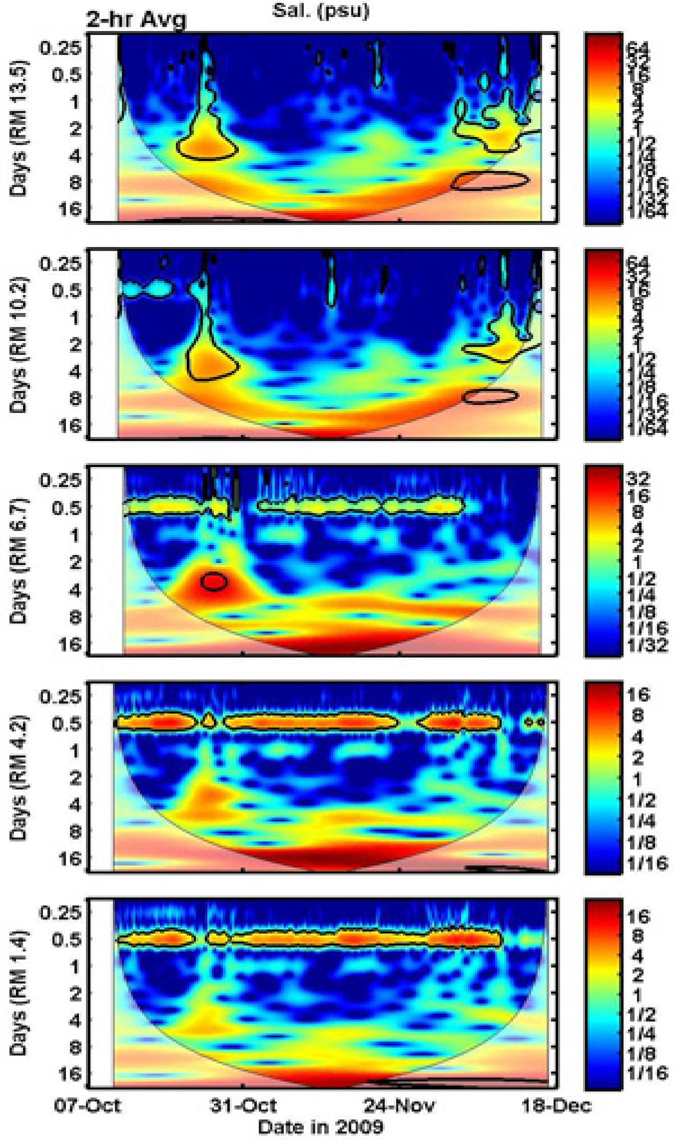
CWTs

- Spring-neap tidal signal strongest for TSS and Velocity
- Semi-diurnal tidal frequency significant for TSS and Velocity
- Semi-diurnal tidal signal strongest for Depth
- Semi-diurnal tidal frequency significant for Salinity at and downstream of RM 6.7
- Precipitation events in October and December significant for Salinity at and upstream of RM 6.7
- Storm event salinity periodicities strongest at 3 days





Correlation?



Wavelets on Multiple Series

- Cross Wavelet Transforms (XWTs) and Wavelet Coherence (WTC) identify the relationships between (e.g. TSS and salinity) and isolate periods when these relationships are statistically significant

- XWTs reveal areas with common high power

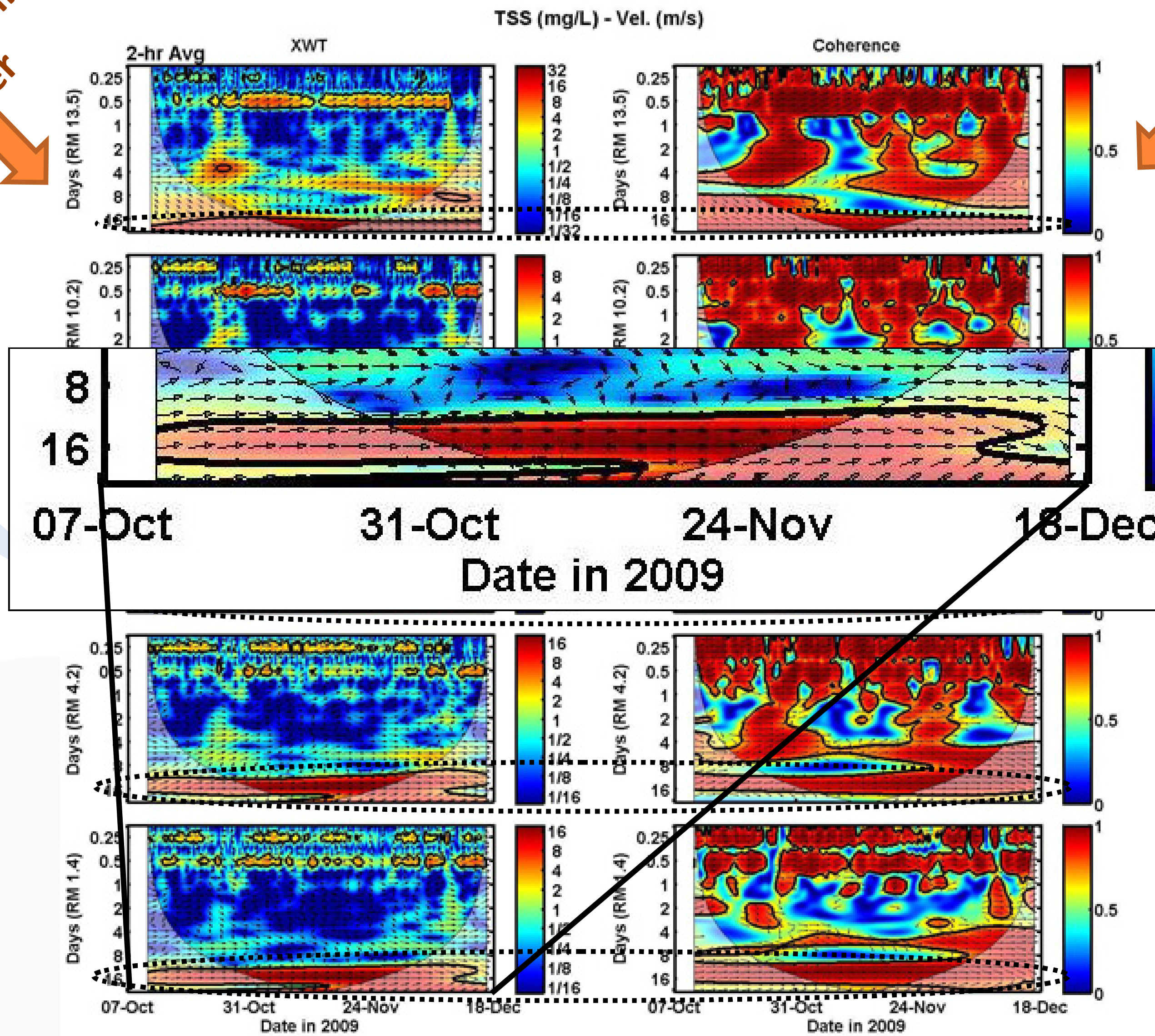
$$W^{XY} = W^{X} W^{Y*}$$

- WTC is similar to a traditional correlation coefficient localized in time frequency space

Results – XWTs & WTCs

TSS and Velocity show high common power

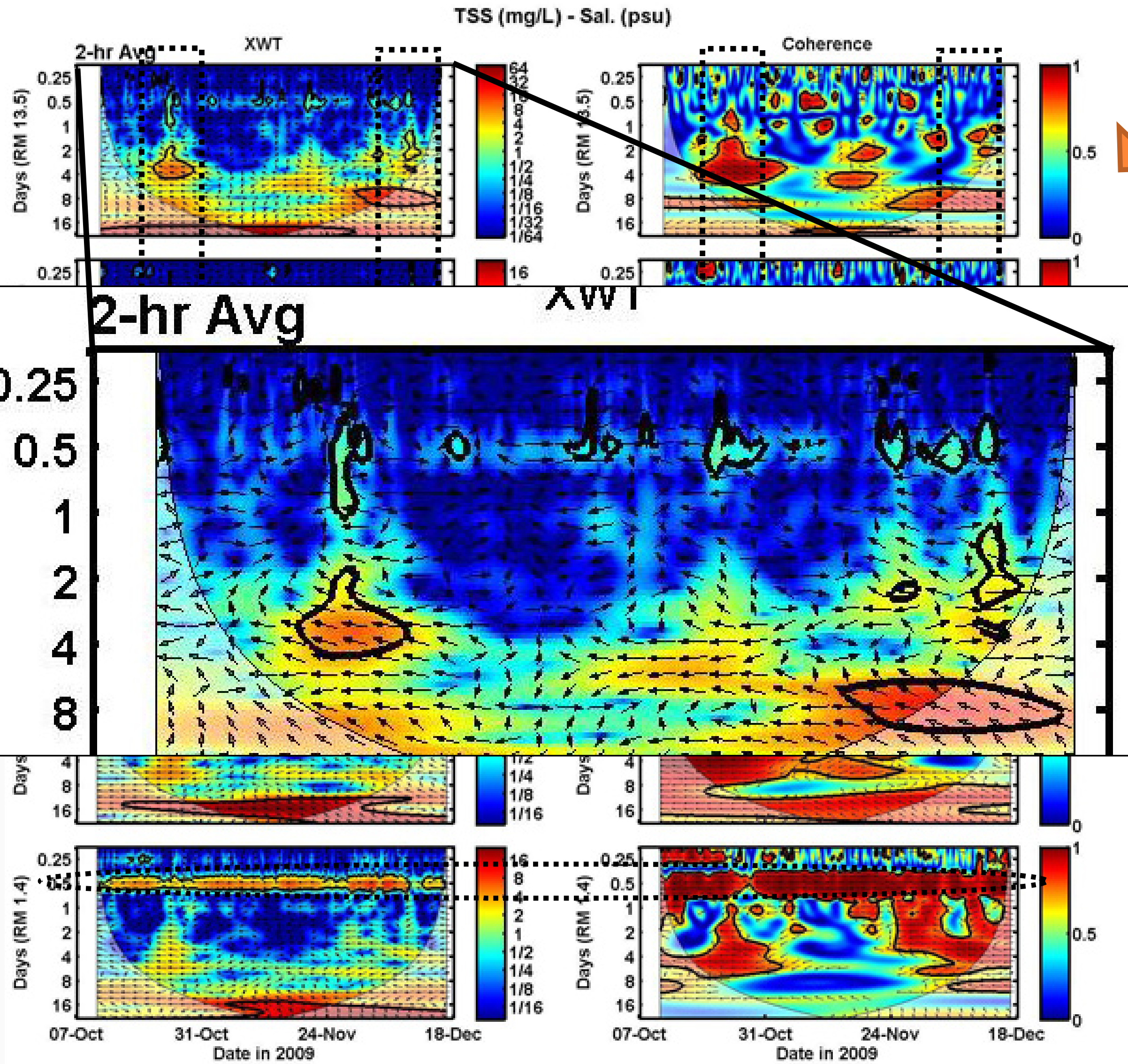
TSS and Velocity co-vary



Results – XWTs & WTCs

TSS and Salinity show high common power

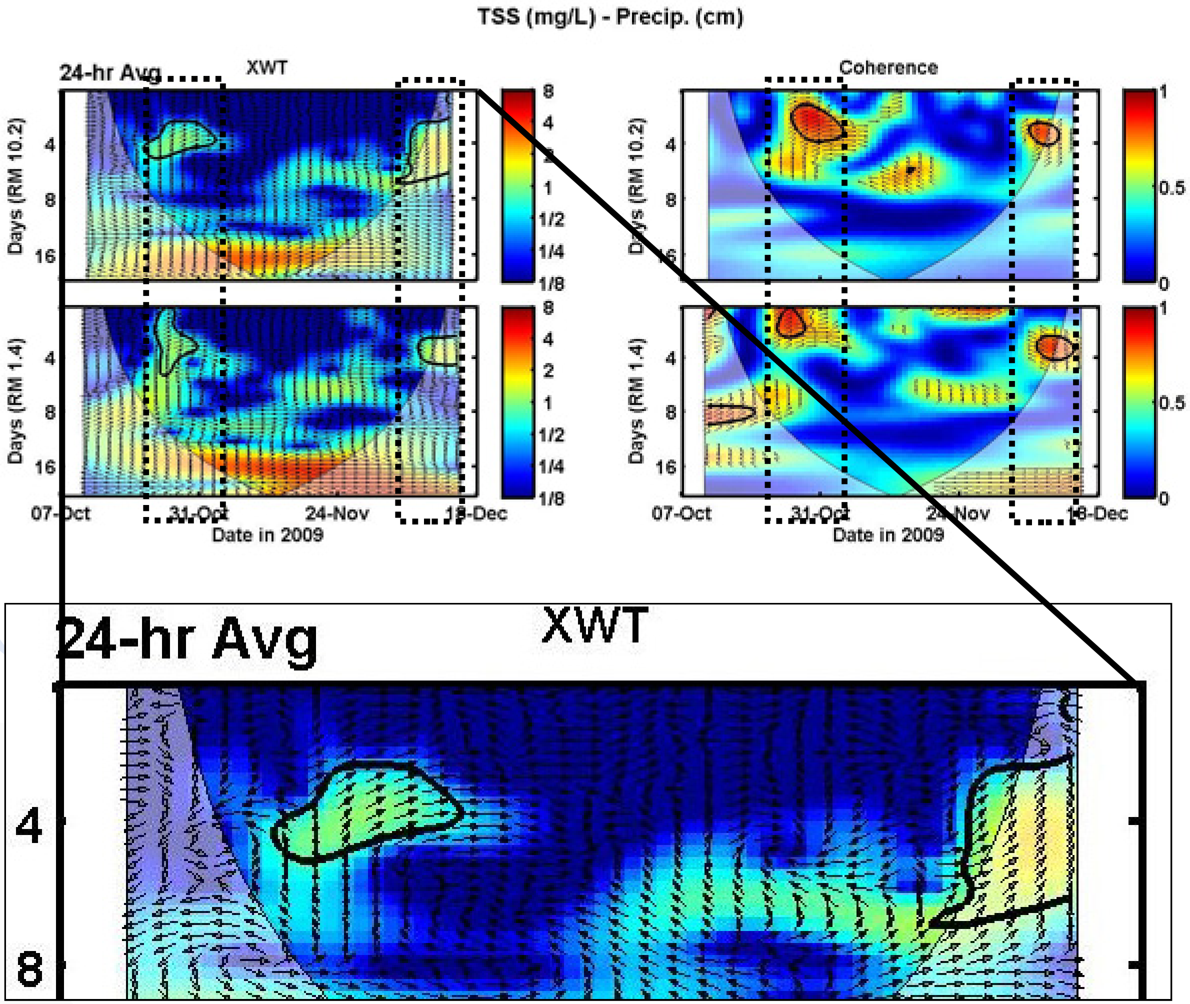
TSS and Salinity co-vary



Results – XWTs & CWTs

TSS and Precip. show high common power

TSS and Precip. co-vary



Summary

XWTs & WTCs

TSS and Salinity exhibit high common power and are coherent at:

- Semi-diurnal tidal periodicity (0.5-day)

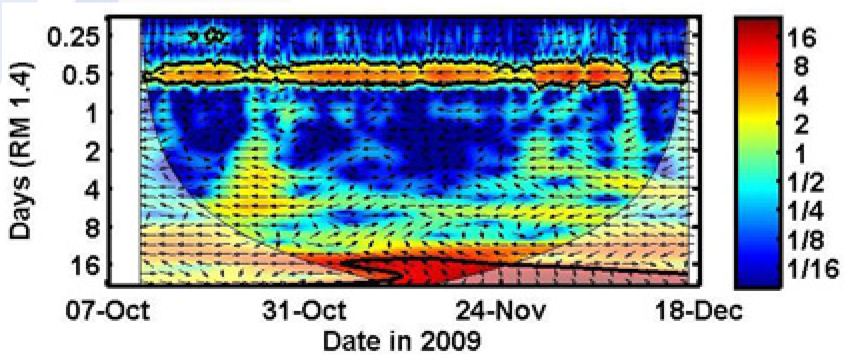
  RMs 6.7, 4.2, and 1.4 only

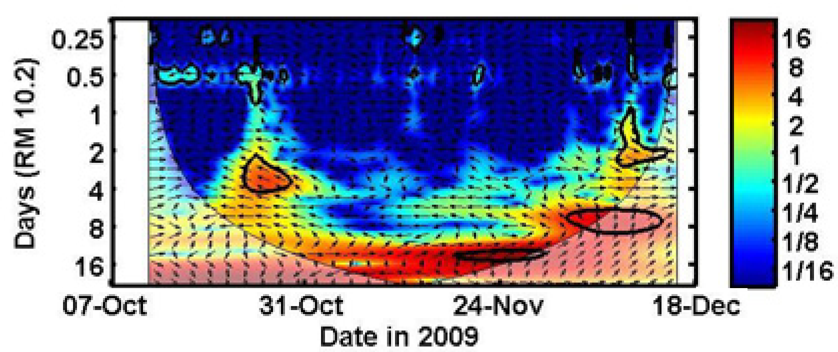
  High salinity (flood tide) = low TSS, low salinity (ebb) = high TSS

- Significant precipitation events (3-days) in October and December

  RMs 13.5 and 10.2 only, RM 6.7 for October event

  Low salinity (precipitation) = high TSS, high salinity = low TSS





Conclusions

• Wavelet analysis facilitates investigations of relationships between TSS and physical parameters

• Transport variability in sediment along the LPR is influenced by :
(1) Tidal fluctuations originating at the mouth of the river
Semi-diurnal (0.5-day): Low and ebb tide = high TSS
Spring-neap (14-days)
(2) Freshwater flow associated with precipitation
Significant events interrupt tidal signal
October at all RMs (strong down to RM 6.7)
December at RMs 13.5 and 10.2

Acknowledgments

US Environmental Protection Agency – Region 2
Larry Sanford – University of Maryland

References

Grinsted, A., J. C Moore, and S. Jevrejeva. 2004. "Application of the cross wavelet transform and wavelet coherence to geophysical time series." *Nonlin. Proc. in Geophys. 11*: 561-566.

http://www.pol.ac.uk/home/research/waveletcoherence/
Wavelet coherence Matlab toolbox.

http://www.amara.com/current/wavelet.html
Nice overview and very extensive reference list.

http://polyvalens.pagesperso-orange.fr/clemens/wavelets/wavelets.html
Easy to understand wavelet tutorial.