Reliability applications of spatio-temporal H_s model

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- Estimation of expected fatigue damage
  - H_s variability over globe
  - Velocity of H_s contours - movement of storms
- Extreme conditions - 100 years encountered H_s
- Uncertainty in fatigue damage prediction and simulation of H_s
  - Correlation of H_s at a fixed location (buoy).
  - Correlation of H_s in space (satellite).
  - Correlation in space an time - along a ships rout
- Summary

Acknowledgments

My interest in studying significant wave height (H_s) variability started around year 2000 with the EU project COMCISS:

Conveying Metocean Knowledge Improvements on to Shipping Safety

This presentation could have the title. Since then I worked, discussed the problem with many colleagues:

Ailliot P., Andersson C., Baxevani A., Borgel C., Brodtkorb PA., Caires S., Lindgren G.,Mao W., Monbet V., Olagnon M., Podgórski K., Prevosto M., Rydén J., Sjö E., Tual L., Wilson, R.J.

I acknowledge their contributions to the presentation.

Fatigue damage - crack initiation and grows

At time t ship, sailing with velocity $\mathbf{V}_{sh}$, encounters sea with significant waves H_s moving with velocity $\mathbf{V}$. Then the fatigue damage of a ship detail grows with a rate

$$d(t) \approx \frac{0.47\, C^3}{\alpha} \left(\frac{H_s^{2.5}}{3.75} - \frac{2\pi}{g} \frac{< \mathbf{V}_{sh}, \mathbf{V} >}{||\mathbf{V}||} \frac{H_s^2}{3.75^2} \right).$$

Given a shipping rout, that takes T hours to sail, the accumulated damage D is

$$D = \int_0^T d(t)\, dt \approx \sum d(t_i) \Delta t.$$

The most important parameters for evaluation of risk for fatigue are the expected damage and its coefficient of variation.

To estimate the parameters one needs shipping rout; description of variability of significant wave heights, ship speed and heading.

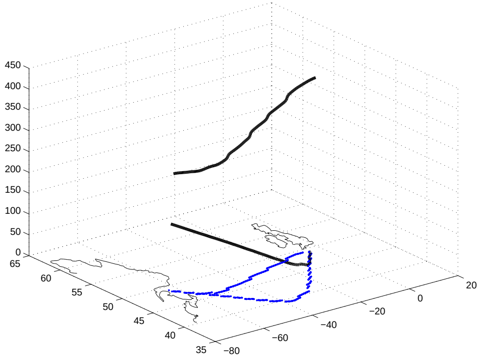

Figure : Illustration of a rout. Blue dots positions $\mathbf{p}(t)$, black dots t in hours, January month. Ships velocity is estimated from the rout. CDF of H_s at a point on the rout and the velocity of H_s are needed.

Model: at any time t and position $\mathbf{p}$, $\ln H_s$ is normally distributed.

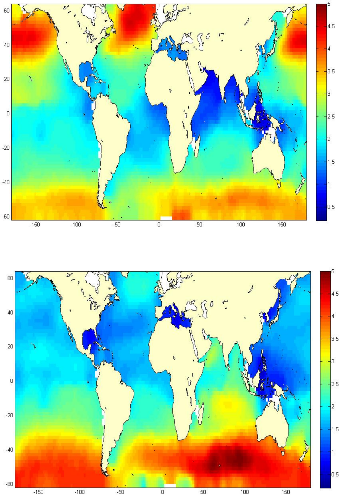

Figure : Estimates of median H_s in February (top plot) and August (bottom plot).

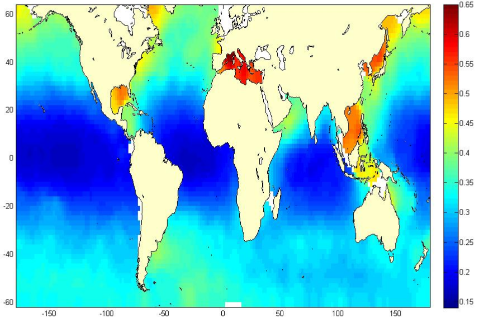

Figure : Estimates of variance of ln H_s, note that the variance is independent of season.

Velocity $\mathbf{V}$ of H_s along ships rout

Longuet-Higgins (1957) was studying models for random, moving surfaces and introduced a concept of velocity. Alternative definition

$$\mathbf{V}(\mathbf{p},t) = \left(-\frac{W_t}{W_x}, -\frac{W_t}{W_y}\right).$$

where $W(\mathbf{p},t) = \ln H_s(\mathbf{p},t)$.

Assuming that W is Gaussian, locally stationary, field with power spectral density $S(\kappa_x,\kappa_y,\omega)$ then the median velocity can be expressed using the spectral moments viz.

$$\mathbf{v}(\mathbf{p},t) = \left(-\frac{\lambda_{101}}{\lambda_{200}}, -\frac{\lambda_{011}}{\lambda_{020}}\right).$$

Here the moments were estimated using ERA 40 data.

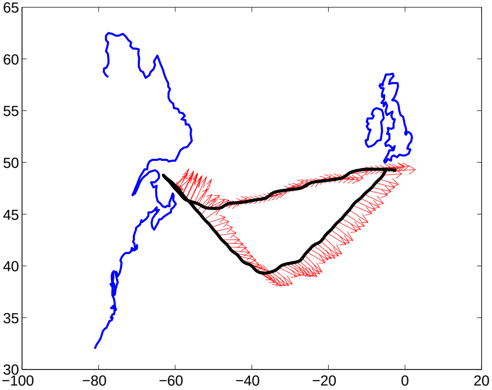

Figure : Estimates of median H_s velocity $\mathbf{v}$ along the rout in January. The term $\frac{<\mathbf{V}_{sh},\mathbf{V}>}{||\mathbf{V}||}$ (the ship speed times minus cosine of heading angle) in damage rate formula will be approximated by $\frac{<\mathbf{V}_{sh},\mathbf{v}>}{||\mathbf{v}||}$.

Having estimated median H_s. velocity of H_s movement and variance of $\ln H_s$, the expected damage rate can be evaluated employing assumed log normality of H_s.

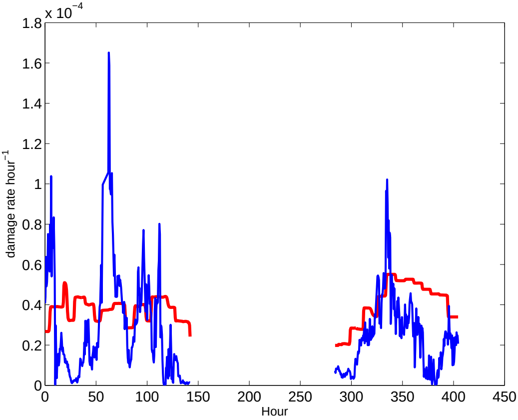

Figure : The expected damage rate red, the observed blue line.This crude analysis indicates that one can expect 1% of total fatigue life to be "consumed". In fact it was abut 0.66% of the life. Warning measurements of H_s are missing -corrected damage is 0.75%!

How big are extreme significant waves along a rout?

Estimation of extreme wave height across the oceans is hampered by lack of data. Buoy and platform data are geographically limited, and though satellite observations offer global coverage, they suffer from temporal sparsity and intermittency, making application of standard methods of extreme value estimation problematical.

A possible strategy in the face of such difficulty is to use extra model assumptions to compensate for lack of data.

Definition: The T years return value w_T is a level which $W(t)$ can exceed during one year with probability $1/T$, viz. with $M(W) = \max_{0\le t\le 1} W(t)$ the return value solves

$$P(M > w_T) = \frac{1}{T}.$$

Rice's method

Let $N(u)$ be the number of upcrossings of the level u by significant waves H_s during unit t one year. Under some assumptions

$$P(M > u) \approx E[N(u)]$$

which can be evaluated by means of Rice's formula if joint variability of significnt wave heightn $H_s(t)$ and its time derivative is known, viz.

$$E[N(u)] \approx \frac{1}{2\pi} \int_0^1 \sqrt{\frac{\lambda_2(t)}{\sigma^2(t)}} \exp\left(-\frac{\ln u - m(t)}{2\sigma^2(t)}\right) \, dt,$$

m, σ^2 are mean and variance of $\ln H_s$ (estimated from satellite data).

It can be shown that $\pi\sqrt{\sigma^2/\lambda_2}$ is approximately the average period a storm is experienced when sailing the rout.[1]

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[1] $\lambda_2(t)$ is a variance of derivative of $\ln H_s(t)$ which is hard to estimated from satellite data

Estimation of λ_2 - correlation structure of $\ln H_s(t)$.

Denote by $r(s)$ the covariance between $\ln H_s(t)$ and $\ln H_s(t+s)$ then

$$\lambda_2 = -r''(0).$$

How to estimate $r(s)$?

Encountered H_s depends on rout, ships speed,movement of storms, development of new waves (wind).

- H_s measured by a buoy is an encountered H_s by vessel moving with a zero speed
- H_s observed from satellite corresponds to encountered significant waves by a vessel moving with almost "infinite" speed.
- H_s encountered by a vessel is between the two extremes

Variability of $\ln H_s(t)$ at fixed position.

Many models for covariance of H_s are proposed in literature. Here we use

$$r(s) = C(\ln H_s(t), \ln H_s(s+t)) = \sigma^2 e^{-\frac{s^2}{2T^2}} e^{-\frac{|s|}{2C}}, \quad 0 \le s \le 10, [h].$$

- The first factor is related to movement of storms while the second represents AR innovations.[2]
- Obviously $r''(0) = \infty$ and Rices method can not be used. Solution: smooth data or covariance $r(s)$. For example let

$$r(s) \approx \sigma^2 e^{-\frac{s^2}{2\tau^2}}, \qquad \tau = -T^2/2C + \sqrt{T^4/4C^2 + T^2}.$$

- Since $\lambda_2 = -r''(0) = \sigma^2/\tau^2$ we have that $\tau = \sqrt{\sigma^2/\lambda_2}$ and hence $\pi\tau$ is the average period H_s stays above the median.

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[2] $e^{-\frac{|s|}{2C}}$ is correlation of Ornstein-Uhlenbeck process which sampled is AR(1).

Variability of $\ln H_s$ in space

- Many authors have reported that H_s recorded along satellite tracks are build up of variability in two scales and that the covariance has (locally) a form of a Gauss function.
- This leads us to the following local covariance function between $\ln H_s(\mathbf{p})$ and $\ln H_s(\mathbf{p}')$

$$C(\mathbf{p},\mathbf{p}') = \sigma^2 \left(p e^{-\frac{||\mathbf{p}-\mathbf{p}'||^2}{2\,L_s^2}} + (1-p) e^{-\frac{||\mathbf{p}-\mathbf{p}'||^2}{2\,L_f^2}} \right) + \sigma_e^2 e^{-\frac{||\mathbf{p}-\mathbf{p}'||^2}{2\,L_e^2}},$$

  where $||\mathbf{p}-\mathbf{p}'|| \leq 4$ degrees.[3]
- Here L_s is the memory length of the "smooth component" (about two to four degrees) L_f is the memory length of fast component (approx. between half and one degree) while L_e is memory of colored noise. In what follows $p = 1$ and $\sigma_e^2 = 0$.

[3] Statistical tests resulted that an isotropic covariance $C(\mathbf{p},\mathbf{p}')$, where $\mathbf{p}$, and $\mathbf{p}'$ are in corrected degrees, could not be rejected.

The parameters L_s, L_f, L_e, p and σ_e^2 are estimated using satellite data.

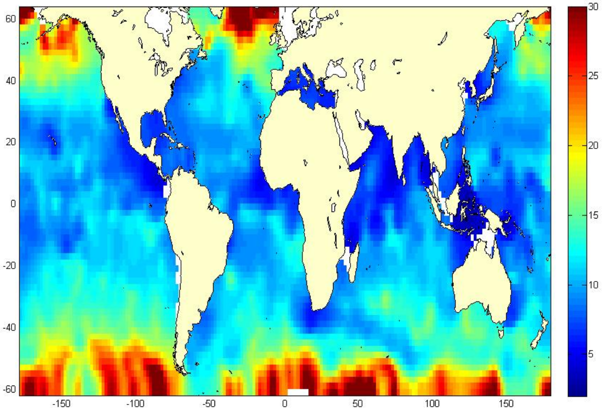


Figure : Estimates of $\pi \cdot L_s$ in January month. The parameter $\pi \cdot L_s$ is a measure of space size of a storm in degrees.

Local correlation of $\ln H_s(t)$ encountered by a vessel

To combine space and time sea state variability we assume that encountered H_s field moves with speed $v = ||\mathbf{V}_{hs} - \mathbf{V}||$. It also constantly changes, which is modeled by AR(1) scheme. Hence:

- Then covariance $r(s) = C(\ln H_s(t), \ln H_s(t+s))$ is

$$r(s) = \sigma^2 e^{-\frac{(vs)^2}{2L_s^2}} e^{-\frac{|s|}{2C}}, \quad 0 \le s \le 10[h].$$

- For a buoy $v = ||\mathbf{V}_{hs}||$ and $r(s)$ coincides with previously discussed covariance ($T = L_s/v$).
- Again, since $r''(0) = \infty$, in order to apply Rices method, the correlation has to be smoothed

$$r(t) \approx e^{-\frac{t^2}{2\tau^2}}, \quad \tau = -(L_s/v)^2/2C + \sqrt{(L_s/v)^4/4C^2 + (L_s/v)^2}.$$

Now $\tau = \sqrt{\sigma^2/\lambda_2}$ is average time a ship is in a storm.

Crossings

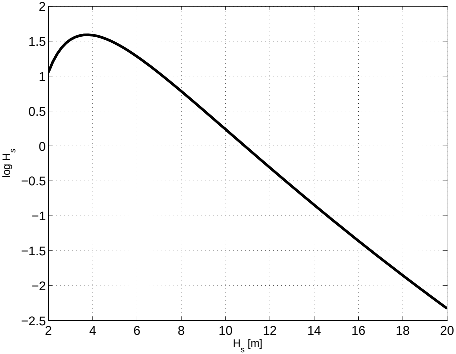


Figure : Expected number of crossings of level u by $H_s(t)$ during a year.

Here yearly shipping is equivalent to seven passages over Atlantic. Then the encountered 100 years H_s is about 18.5 meters (possible captain decisions/routing to avoid storms are neglected.)

Validation: for buoy 44005 the 100 years H_s predicted using the model is 16.2 meter. In literature a value of 16.6 meters were reported.

Model parameters encountered on the rout

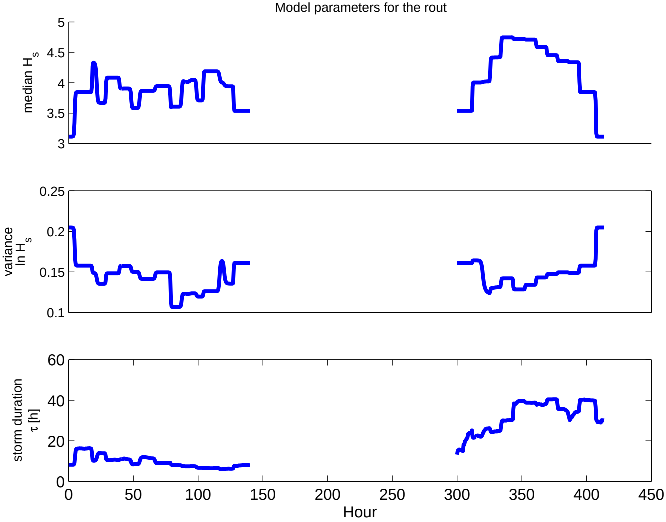


Figure : Parameters of the model along the rout; median H_s; variance of $\ln H_s$; τ correlation length.

Variance of damage - simulation of $H_s(t)$

In order to simulate a sequence of encountered H_s, estimate the missing values, or compute the variance of the accumulated fatigue damage, one needs the correlation structure of $\ln H_s(t)$ along the entire rout.

Until now we modeled H_s as locally stationary log-normal process with covariance structue $r(s) \approx \sigma^2 \exp(-s^2/\tau)$. As shown in the figure median H_s, σ^2 and τ slowly changes with time, i.e. $\ln H_s(t)$ is non-stationary pr.

Using time variable moving average process one derive a Gaussian model for $\ln H_s(t)$ having the covariance function between $\ln H_s(t)$ and $\ln H_s(t+s)$ given by

$$C(t,s) = \sigma(t)\sigma(s)\sqrt{\frac{2\tau(t)\tau(s)}{\tau(t)^2+\tau(s)^2}}\, e^{-\frac{(s-t)^2}{2(\tau(t)^2+\tau(s)^2)}}.$$

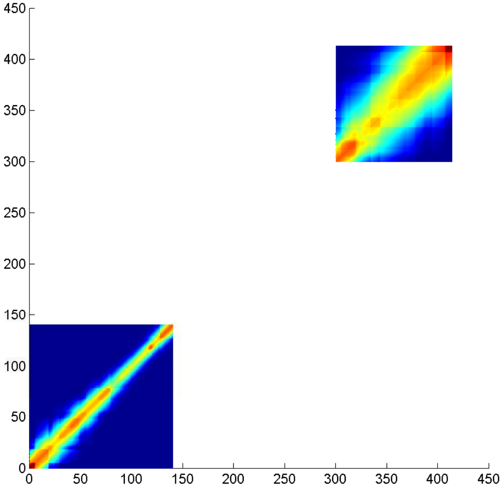
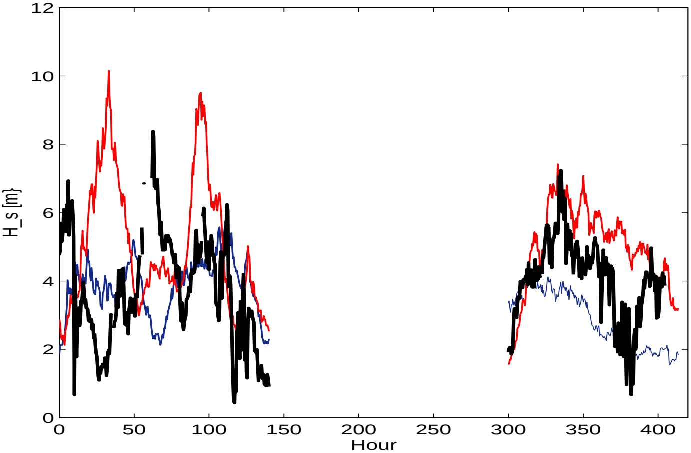


Figure : The covariance $C(t, s)$ between logarithms of H_s along the rout (t, s in hours). Time spend in harbor (about 150 hours) made H_s on the rout to America independent of H_s on the way back to Europe.



Observed H_s black line, two simulated H_s histories red and blue lines.

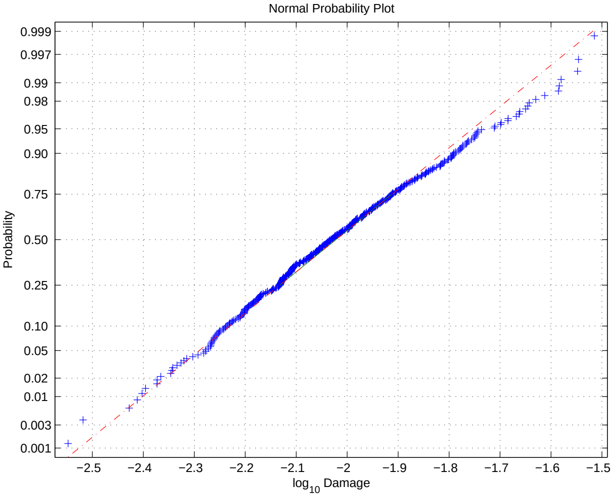


Figure : Logarithms of simulated 400 damages accumulated on the rout plotted on normal probability paper. Mean damage 0.01 and coefficient of variation 0.43.

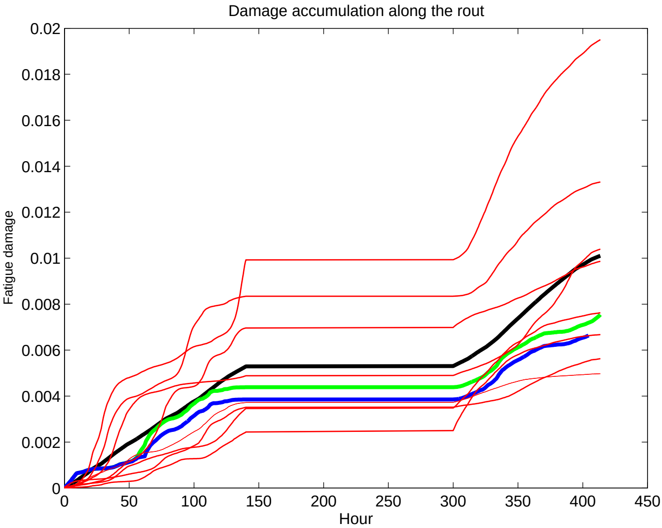


Figure : Simulated damage accumulation processes red. The expected accumulation - black. Observed accumulation of the damage (some H_s values are missing) - blue. Observed damage accumulation after including predicted missing H_s - green.

Summary

- A model for variability of encountered significant wave height was presented. The model can be used for any rout on globe.
- Parameters are estimated using satellite data (spatial variability); ERA 40 data (median velocities of of H_s field). Buoys (time dynamic - AR innovations in H_s field).
- Although the estimation of model parameters was tedious and took long time. The use of it is simple and computations very fast.
- Model can be a useful tool in estimation of risks for fatigue failure of ship details and in quantifying the uncertainties.

Thank You For Attention!